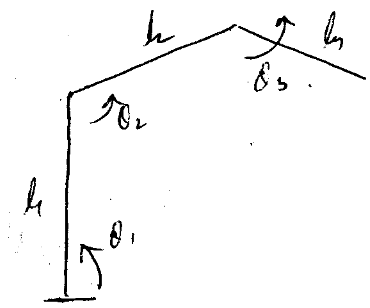
Bernard

Q.2



Dello

The link parameters are given as below:

$$A_1 = \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ s_1 & -c_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

I need the inv kinematics, not the forward

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then,

$$T_0^1 = A_1$$

$$T_0^2 = A_1 A_2$$

$$T_0^3 = A_1 A_2 A_3$$

$$= \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 l_3 \\ s_1 & 0 & -c_1 & c_1 d_3 \\ 0 & -1 & 0 & l_1 + l_2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -s_1 l_3 \\ c_1 l_3 \\ l_1 + l_2 \end{bmatrix}$$

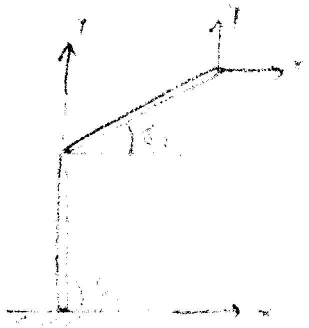
then Solve
for Joint
parameters

Need
the inv.
Kinematics

$\theta_1 = f_1(x, y, z)$

$\theta_2 = f_2(x, y, z)$

etc



The A-matrices are determined with

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link parameters can be derived

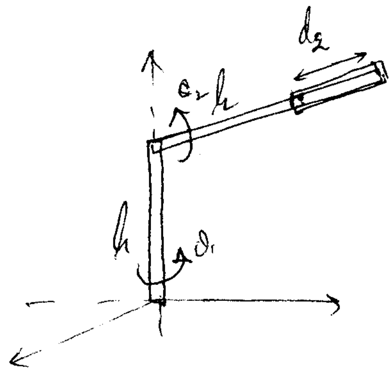
link	a_i	α_i	d_i	θ_i
1	a_1	90	0	θ_1
2	a_2	0	0	θ_2

Then, $T_0^2 = A_1 A_2$

$$= \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_1c_2 \\ s_{12} & c_{12} & 0 & a s_1 + a_2 s_1 s_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(1.)



Denavit-Hartenberg representation depends on 4 different homogeneous transformation.

DH-convention will be.

link	a_i	α_i	d_i	θ_i
1	0	0	0	0
2	0	180	d_2	0
3	0	0	0	0

Now,

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then $T_0^3 = A_1A_2A_3$.

$$T_0^3 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ c_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

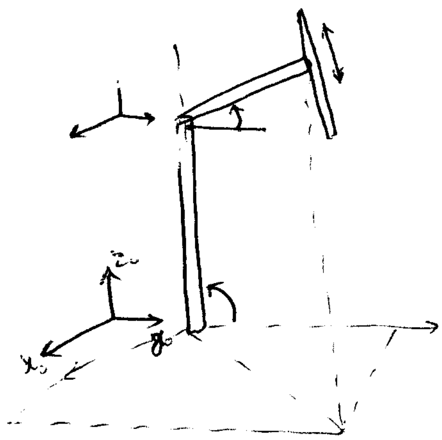
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12} + s_{12} & -c_{12} + s_{12} & 0 & a_1c_1 + a_2c_2 \\ s_{12} - c_{12} & -s_{12} - c_{12} & 0 & a_1s_1 + a_2s_2 \\ 0 & 0 & -1 & -d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_1c_1 + a_2c_2 \\ a_1s_1 + a_2s_2 \\ -d_2 \\ 1 \end{bmatrix}$$

(4)



$$A_1 = \begin{bmatrix} c_1 & 0 & -c_2 & 0 \\ s_1 & 0 & s_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 A_2 A_3$$

$$= \begin{bmatrix} C_1 & 0 & -S_2 & 0 \\ s_1 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DH convention.

Link	a_i	α_i	d_i	θ_i
1	0	0	l_1	θ_1
2	0	0	0	0
3	0	180°	d_3	0.

Then.

Now,

$$J(q) = \begin{bmatrix} z_0(o_3 - o_0) & z_1(o_3 - o_1) & z_2(o_3 - o_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

Then,
$$o_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$o_1 = \begin{pmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{pmatrix}$$

$$o_2 = \begin{pmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{pmatrix}$$

$$o_3 = \begin{pmatrix} a_1 c_1 + a_2 c_{12} + a \\ \end{pmatrix}$$