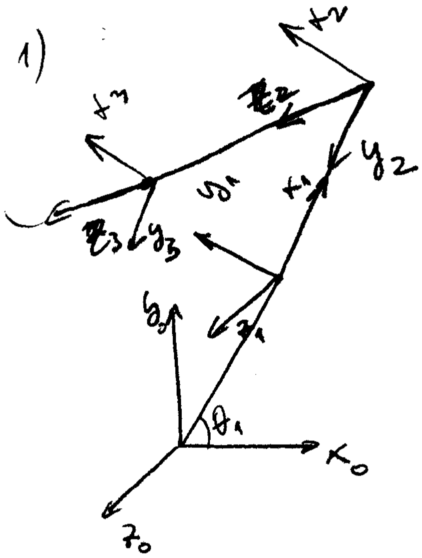
ANDREW ROSCA

1)



	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	90
3	0	0	l_3	0

?X

Redo as HW

no credit

$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & l_1 C_1 \\ S_1 & C_1 & 0 & l_1 S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = \begin{bmatrix} -S_1 & -C_1 & 0 & C_1 l_1 - l_2 S_1 \\ C_1 & -S_1 & 0 & C_1 l_2 + l_1 S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

then solve equations

$$A_1 A_2 A_3 = \begin{bmatrix} -S_1 & -C_1 & 0 & -l_1 S_1 + C_1 l_1 \\ C_1 & -S_1 & 0 & C_1 l_2 + l_1 S_1 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$= -9 \le x \le 9$

$= -9 \le y \le 9$

$0 \le z \le 3$

~~$0 \le y \le 9$ $(l_1 + l_2)$~~

~~$0 \le x \le 9$~~

~~$0 \le z \le 3$ (l_3)~~

$x = C_1 l_1 - l_1 S_1$

$y = C_1 l_2 + l_1 S_1$

$z = l_3$

$$R = \begin{bmatrix} -S_1 & -C_1 & 0 \\ C_1 & -S_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_1 = \frac{x + l_1 s_1}{l_1}$$

$$s_1 = \frac{y - c_1 l_2}{l_1} = \frac{y - \frac{x + l_1 s_1}{l_1} l_2}{l_1} = \frac{(y l_1 + l_1 s_1) l_2}{l_1^2}$$

$$s_1 l_1^2 - l_1 l_2 s_1 = y l_1 l_2 \Rightarrow s_1 = \frac{y l_1 l_2}{(l_1^2 - l_1 l_2)}$$

$$\Rightarrow \theta_1 = \arcsin \frac{y l_1 l_2}{l_1^2 - l_1 l_2}$$

$$\left(c_1 = \frac{x + l_1 s_1}{l_1} = \frac{x + \cancel{l_1} \frac{y - c_1 l_2}{\cancel{l_1}}}{l_1} = \frac{x \cancel{l_1} + \cancel{l_1}(y - c_1 l_2)}{l_1^{\cancel{2}}} = \right.$$

$$= \frac{x + y - c_1 l_2}{l_1} \Rightarrow c_1 (1 + l_2) = \frac{x + y}{l_1}$$

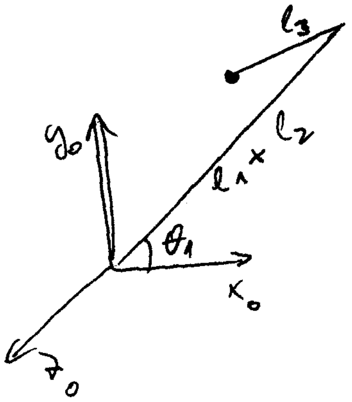
$$\left. c_1 = \frac{x + y}{l_1 (1 + l_2)} \qquad \theta_1 = \arccos \frac{x + y}{l_1 (1 + l_2)} \right)$$

$l_3 = z$ $\qquad$ l_1 – fixed (constant)

$$\theta_1 = \arcsin \frac{y l_1 l_2}{l_1^2 - l_1 l_2} \qquad \arccos \frac{x + y}{l_1 (l_2 + 1)} = \arcsin \frac{y l_1 l_2}{l_1^2 - l_1 l_2}$$

$$\Rightarrow l_2$$

<u>geometrically</u>



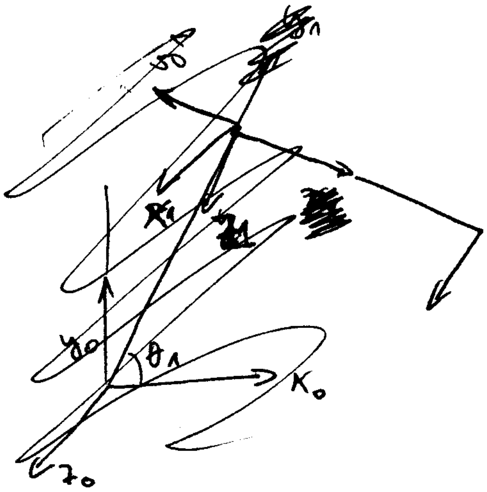
$l_3 = z.$

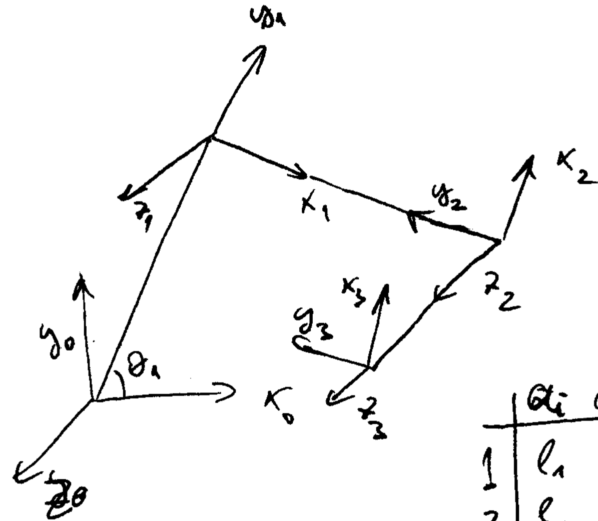
$$\left(l_1 + l_2 = \arcsin \frac{x}{l_1 + l_2} \right) \qquad (l_1 \text{ constant})$$

$$l_2 = \sqrt{x^2 + y^2} - l_1$$

$$\theta_1 = \arcsin \frac{x}{l_1 + l_2} = \frac{x}{\sqrt{x^2 + y^2}}$$

2)





	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	90
3	0	0	l_3	0

$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & C_1 l_1 \\ S_1 & C_1 & 0 & S_1 l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = \begin{bmatrix} -S_1 & -C_1 & 0 & -S_1 l_2 + C_1 l_1 \\ C_1 & -S_1 & 0 & C_1 l_2 + S_1 l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} -S_1 & -C_1 & 0 & -S_1 l_2 + C_1 l_1 \\ C_1 & -S_1 & 0 & C_1 l_2 + S_1 l_1 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = C_1 l_1 - S_1 l_2$$
$$y = C_1 l_2 + S_1 l_1$$
$$z = l_3$$

$$-3 \le x \le 3$$
$$-3 \le y \le 3$$
$$0 \le z \le 3$$

$$C_1 = \frac{x + S_1 l_2}{l_1} = \cancel{\ldots}$$

$$y = \frac{x + S_1 l_2}{l_1} l_2 + S_1 l_1$$

$$y l_1 = x + S_1 l_2^2 + S_1 l_1^2$$

$$S_1 (l_1^2 + l_2^2) = y l_1 - x$$

$$\theta_1 = \arcsin \frac{y l_1 - x}{l_1^2 + l_2^2} \quad \left(\begin{matrix} l_1 - \text{constant} \\ = 5 \end{matrix} \right)$$

$$l_3 = z$$

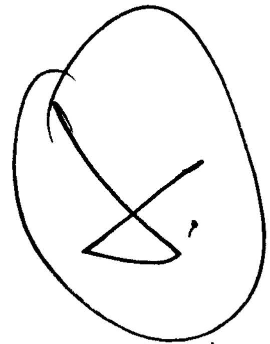
$$l_2 \Leftarrow \arcsin \frac{y l_1 - x}{l_1^2 + l_2^2} = \arccos \frac{\ldots}{\ldots}$$

geometrically

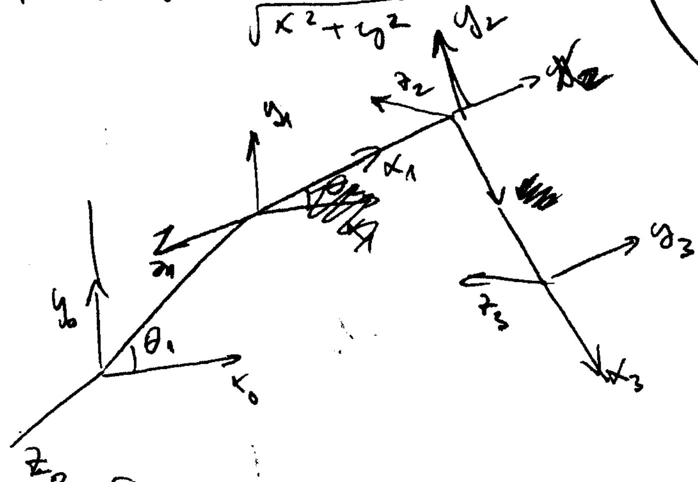
$l_3 = z$

$l_2 = \sqrt{-l_1^2 + \sqrt{x^2+y^2}}$

$\theta_1 = \arcsin \frac{x}{\sqrt{x^2+y^2}}$



3)



	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	θ_2
3	l_3	0	0	-90

$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & C_1 l_1 \\ S_1 & C_1 & 0 & S_1 l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & C_2 l_2 \\ S_2 & C_2 & 0 & S_2 l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_1 A_2 = \begin{bmatrix} C_{12} & -S_{12} & 0 & C_1 l_1 + C_2 l_2 \\ S_{12} & C_{12} & 0 & S_1 l_1 + S_2 l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

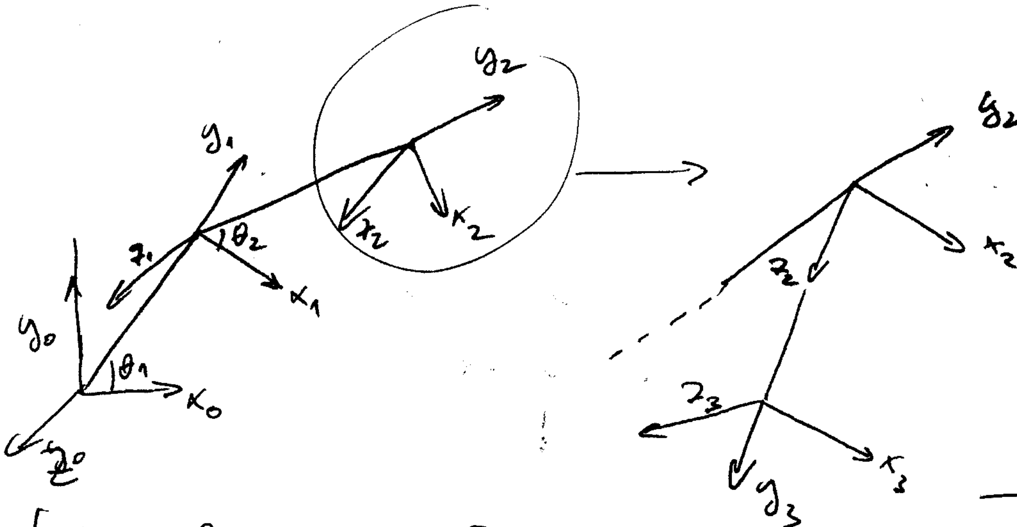
$$A_1 A_2 A_3 = \begin{bmatrix} -S_{12} & -C_{12} & 0 & S_1 l_1 + S_2 l_2 \\ C_{12} & -S_{12} & 0 & -C_1 l_1 - C_2 l_2 - l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$z = 0$

$x = S_1 l_1 + S_2 l_2$

$y = -(C_1 l_1 + C_2 l_2 + l_3)$

3)



	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	θ_2
3	0	90	l_3	0

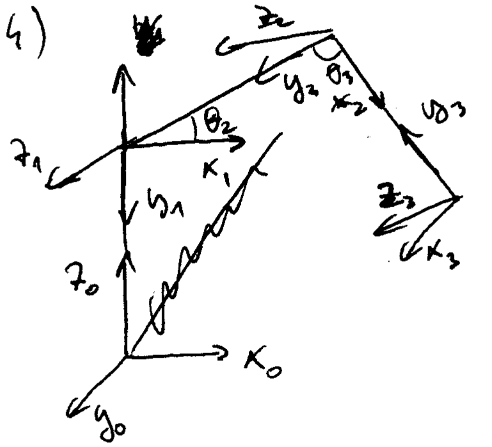
$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & C_1 l_1 \\ S_1 & C_1 & 0 & S_1 l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & C_2 l_2 \\ S_2 & C_2 & 0 & S_2 l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\sin\alpha = 1$
$\cos\alpha = 0$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_1 A_2 = \begin{bmatrix} C_{12} & -S_{12} & 0 & C_1 l_1 + C_2 l_2 \\ S_{12} & C_{12} & 0 & S_1 l_1 + S_2 l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} C_{12} & -S_{12} & 0 & C_1 l_1 + C_2 l_2 \\ 0 & 0 & -1 & 0 \\ S_{12} & C_{12} & 0 & l_3 + \\ & & & \end{bmatrix}$$

4)



	a_i	α_i	d_i	θ_i
1	0	90	l_1	θ_1
2	l_2	0	0	θ_2
3	l_3	0	0	θ_3

$$A_1 = \begin{bmatrix} c_1 & 0 & 0 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & C_2 l_2 \\ S_2 & C_2 & 0 & S_2 l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & C_3 l_3 \\ S_3 & C_3 & 0 & S_3 l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 A_3 = \begin{bmatrix} C_{23} & -S_{23} & 0 & C_2 l_2 + C_3 l_3 \\ S_{23} & C_{23} & 0 & S_2 l_2 + S_3 l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} C_1 C_{23} & -S_{23} C_1 & 0 & C_1 (C_2 l_2 + C_3 l_3) \\ S_1 C_{23} & -S_{23} S_1 & -C_1 & S_1 (C_2 l_2 + C_3 l_3) \\ 0 & 0 & 1 & l_1 + S_2 l_2 + S_3 l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$x = C_1 (C_2 l_2 + C_3 l_3)$ $\qquad C_1 = \dfrac{x}{C_2 l_2 + C_3 l_3}$

$y = S_1 (C_2 l_2 + C_3 l_3)$

$z = l_1 + S_2 l_2 + S_3 l_3$