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Redo & Expand ($\frac{dV}{dt}$, inv. velocity) inv. kinematics

Quiz

1)

Kinematics (Forward):

$$x = l_1 \cos\theta_1 + l_2 \cos\theta_2$$
$$y = l_1 \sin\theta_1 + l_2 \sin\theta_2$$

D-H parameter table:

Link	α	a	d	θ
1	0	0	l_1	θ_1
2	0	0	l_2	θ_2

$$T_0^1 = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

$$T_0^2 = \begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2 \\ -\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & l_1 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_0^2 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & l_2 \\ -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse Kinematics:

$$\begin{bmatrix} c_1c_2 - s_1s_2 & c_1s_2 + s_1c_2 \\ -s_1c_2 - c_1s_2 & -s_1s_2 + c_1c_2 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

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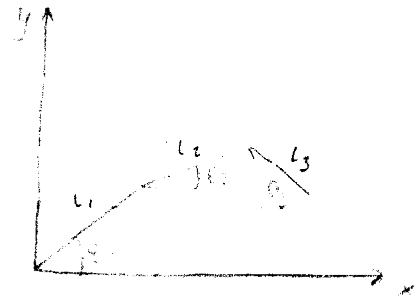

D-H Table

Link	α	a	d	θ
1	0	0	l_1	θ_1
2	0	0	l_2	θ_2
3	0	0	l_3	θ_3

$$T_0^1 = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 \\ -\sin\theta_3 & \cos\theta_3 \end{bmatrix}$$

$$T_0^{3} = \begin{bmatrix} C_1 C_2 C_3 - C_3 S_1 S_2 + S_1 S_2 S_3 - C_1 S_2 S_3 & C_1 S_2 C_3 + S_1 C_2 C_3 - S_1 S_2 S_3 - C_1 C_2 S_3 \\ -C_1 C_2 S_3 + S_1 S_2 S_3 - S_1 S_2 C_3 - C_1 C_3 S_2 & -C_1 S_2 S_3 - S_1 C_2 S_3 - S_1 S_2 C_3 + C_1 C_2 C_3 \end{bmatrix}$$

Inverse Kinematics

$$\begin{bmatrix} C_1 C_2 C_3 - S_1 S_2 C_3 - S_1 S_2 S_3 - C_1 S_2 S_3 & C_1 S_2 C_3 + S_1 C_2 C_3 - S_1 S_2 S_3 - C_1 C_2 S_3 \\ -C_1 C_2 S_3 + S_1 S_2 S_3 - S_1 S_2 C_3 - C_1 C_3 S_2 & -C_1 S_2 S_3 - S_1 C_2 S_3 - S_1 S_2 C_3 + C_1 C_2 C_3 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

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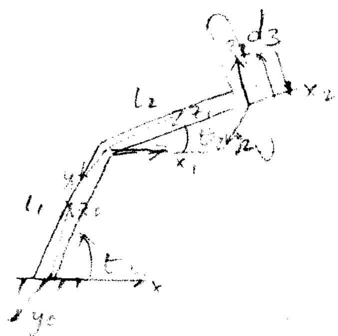
3)



D-H Table

Link	α	a	d	θ
1	0	l_1	0	θ_1
2	0	l_2	0	θ_2
3	90°	0	d_3	0

$$T_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$T_0^3 = T_0^1 \cdot T_1^2 \cdot T_2^3$$

$$T_0^2 = \begin{bmatrix} c_1c_2 - s_1s_2 & -c_1s_2 - s_1c_2 & 0 & c_1l_2c_2 - s_1l_2c_2 + l_1c_1 \\ s_1c_2 + c_1s_2 & -s_1s_2 + c_1c_2 & 0 & s_1l_2c_2 + c_1l_2s_2 + l_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} c_1c_2 - s_1s_2 & -c_1s_2 - s_1c_2 & 0 & c_1l_2c_2 - s_1l_2c_2 + l_1c_1 \\ s_1c_2 + c_1s_2 & -s_1s_2 + c_1c_2 & 0 & s_1l_2c_2 + c_1l_2s_2 + l_1s_1 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinemitics:

$$\begin{bmatrix} c_1l_2c_2 - s_1l_2c_2 + l_1c_1 \\ s_1l_2c_2 + c_1l_2s_2 + l_1s_1 \\ d_3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Inverse kinematics:

$$\begin{bmatrix} c_1c_2 - s_1s_2 & -c_1s_2 - s_1c_2 \\ s_1c_2 + c_1s_2 & -s_1s_2 + c_1c_2 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

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$z_0 = [0 \quad 0 \quad 1]$
$z_1 = [0 \quad 0 \quad 1]$
$z_2 = [0 \quad 0 \quad 1]$
$z_3 = [0 \quad 0 \quad 1]$

$O_0 = 0$

$$O_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} c_1 l_2 c_2 - s_1 l_2 c_2 + l_1 c_1 \\ s_1 l_2 c_2 + c_1 l_2 s_2 + l_1 s_1 \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} c_1 l_2 c_2 - s_1 l_2 c_2 + l_1 c_1 \\ s_1 l_2 c_2 + c_1 l_2 s_2 + l_1 s_1 \\ d_3 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} c_1 l_2 c_2 - s_1 l_2 c_2 + l_1 c_1 \\ s_1 l_2 c_2 + c_1 l_2 s_2 + l_1 s_1 \\ 0 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} c_1 l_2 c_2 - s_1 l_2 c_2 + \cancel{l_1 c_1} - \cancel{l_1 c_1} \\ s_1 l_2 c_2 - s_1 l_2 s_2 + \cancel{l_1 s_1} - \cancel{l_1 s_1} \\ 0 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} 0 \\ 0 \\ +d_3 \end{bmatrix}$$

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$$J = \begin{bmatrix} c_1 l_2 c_2 - s_1 l_2 c_2 + l_1 c_1 & c_1 l_2 c_2 - s_1 l_2 c_2 & 0 \\ s_1 l_2 c_2 + c_1 l_2 s_2 + l_1 s_1 & s_1 l_2 c_2 - s_1 l_2 s_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

$\dot{X} = J \times \dot{\theta}$ Forward ~~kinematics~~ velocity

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}^{\cdot} = \begin{bmatrix} c_1 l_2 c_2 - s_1 l_2 c_2 + l_1 c_1 & c_1 l_2 c_2 - s_1 l_2 c_2 & 0 \\ s_1 l_2 c_2 + c_1 l_2 s_2 + l_1 s_1 & s_1 l_2 c_2 - s_1 l_2 s_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ 0 \end{bmatrix}$$

$J^{-1} \cdot \dot{X} = \dot{\theta}$ inverse ~~kinematics~~ velocity

$\ddot{X} = J \times \ddot{\theta} + \frac{d(J)}{dt} \dot{\theta}$ Forward ~~velocity~~ acceleration

$\ddot{\theta} = J^{-1} \times \left(\ddot{X} - \frac{d(J)}{dt} \dot{\theta} \right)$ Inverse acceleration

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Forward acceleration:

$$\ddot{X} = J \times \ddot{\theta} + \frac{d(J)}{dt}\dot{\theta}$$

$$= \begin{bmatrix} c_1 l_2 c_2 - s_1 l_2 c_2 + l_1 c_1 & c_1 l_2 c_2 - s_1 l_2 c_2 & 0 \\ s_1 l_2 c_2 + c_1 l_2 s_2 + l_1 s_1 & s_1 l_2 c_2 - s_1 l_2 s_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} -c_2 s_1 l_1 - c_1 s_2 l_1 - c_2 c_1 l_2 - s_1 s_2 l_2 - s_1 l_1 & -s_1 l_2 c_2 - s_2 l_2 c_1 + s_2 s_1 l_2 - c_1 c_2 l_2 & 0 \\ c_2 c_1 l_2 - s_2 s_1 l_2 - s_2 s_1 l_2 + c_2 c_1 l_2 + c_1 l_1 & -s_2 l_2 s_1 + c_1 c_2 l_2 - c_1 s_2 l_2 + c_2 s_1 l_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ 0 \end{bmatrix}$$

$$\ddot{X} = \begin{bmatrix} c_1 l_2 c_2 - s_1 l_2 c_2 + l_1 c_1 & c_1 l_2 c_2 - s_1 l_2 c_2 & 0 \\ s_1 l_2 c_2 + c_1 l_2 s_2 + l_1 s_1 & s_1 l_2 c_2 - s_1 l_2 s_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} c_2 s_1 l_1 - c_1 s_2 l_1 - c_2 c_1 l_2 - s_1 s_2 l_2 - s_1 l_1 & -s_1 l_2 c_2 - s_2 l_2 c_1 + s_2 s_1 l_2 - c_1 c_2 l_2 & 0 \\ c_2 l_2 c_1 - s_1 l_2 s_2 - s_2 s_1 l_2 + c_2 c_1 l_2 + c_1 l_1 & -s_2 l_2 s_1 + c_1 c_2 l_2 - c_1 s_2 l_2 + c_2 s_1 l_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ 0 \end{bmatrix}$$