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ID # 393617

-50

Expand
S, $\frac{d\theta}{dt}$, i vector

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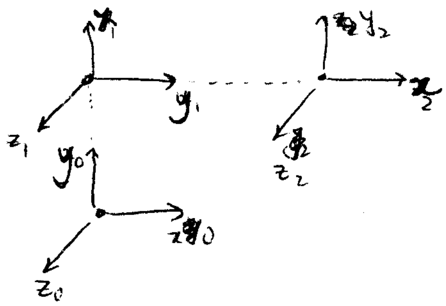
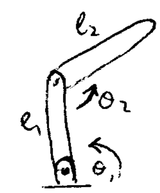
Systematically find the 6 modules : ① kinematics ② inverse kinematics ③ velocity ④ inverse velocity ⑤ Acceleration ⑥ inverse acceleration.

1





2-D

	a	d	α	θ
1	l_1	0	0	θ_1^*
2	l_2	0	0	θ_2^*

* variable

$$A_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & l_1\cos\theta_1 \\ s\theta_1 & c\theta_1 & 0 & l_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$c\theta_1 = \cos\theta_1$
$s\theta_1 = \sin\theta_1$

$$A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^2 = A_1 \cdot A_2 = \begin{bmatrix} c\theta_1 c\theta_2 - s\theta_1 s\theta_2 & -c\theta_1 s\theta_2 - s\theta_1 c\theta_2 & 0 & l_2(c\theta_1 c\theta_2 - s\theta_1 s\theta_2) + l_1 c\theta_1 \\ s\theta_1 c\theta_2 + c\theta_1 s\theta_2 & -s\theta_1 s\theta_2 + c\theta_1 c\theta_2 & 0 & l_2(s\theta_1 c\theta_2 + c\theta_1 s\theta_2) + l_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = l_2(c\theta_1 c\theta_2 - s\theta_1 s\theta_2) + l_1 c\theta_1$$
$$y = l_2(s\theta_1 c\theta_2 + c\theta_1 s\theta_2) + l_1 s\theta_1$$

$$J_0^n = \left[\begin{array}{c} J_v \\ \hline J_\omega \end{array}\right] \begin{array}{l} \rightarrow \text{linear velocity} \\ \rightarrow \text{angular velocity} \end{array}$$

$P_{12} =$ ~~$z_0 \times (O_2 - O_0)$~~

Jacobian, $J(q) = \begin{bmatrix} z_0 \times (O_2 - O_0) & z_1 \times (O_2 - O_1) \\ z_0 & z_1 \end{bmatrix}$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad O_1 = \begin{bmatrix} l_1 c\theta_1 \\ l_1 s\theta_1 \\ 0 \end{bmatrix} ; \quad O_2 = \begin{bmatrix} l_1 c\theta_1 + l_2(c\theta_1 c\theta_2 - s\theta_1 s\theta_2) \\ l_1 s\theta_1 + l_2(s\theta_1 c\theta_2 + c\theta_1 s\theta_2) \\ 0 \end{bmatrix}$$

$$z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 s\theta_1 - l_2(s\theta_1 c\theta_2 + c\theta_1 s\theta_2) & -l_2(s\theta_1 c\theta_2 + c\theta_1 s\theta_2) \\ l_1 c\theta_1 + l_2(c\theta_1 c\theta_2 - s\theta_1 s\theta_2) & l_2(c\theta_1 c\theta_2 - s\theta_1 s\theta_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\dot{X} = J \cdot \dot{\theta}$$

$$\ddot{X} = J\ddot{\theta} + \frac{dJ}{dt} \cdot \dot{\theta}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \dot{\theta}_1 - l_2 \dot{\theta}_1 (s_1 c_2 + c_1 s_2) - l_2 \dot{\theta}_2 (s_1 c_2 + c_1 s_2) \\ l_1 c_1 \dot{\theta}_1 + l_2 \dot{\theta}_1 (c_1 c_2 - s_1 s_2) + l_2 \dot{\theta}_2 (c_1 c_2 - s_1 s_2) \\ 0 \\ 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

Where's the rest $\frac{dJ}{dt}$ in acceleration? -8%

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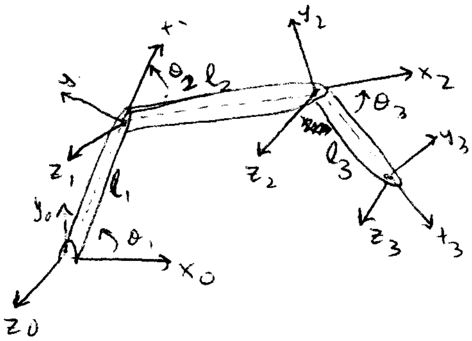
(2)



	a	d	α	θ
1	l_1	0	0	θ_1^*
2	l_2	0	0	θ_2^*
3	l_3	0	0	θ_3^*

* variable

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & l_3 c_3 \\ s_3 & c_3 & 0 & l_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 \cdot A_2 \cdot A_3 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{12} & c_{12} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c_3 & -s_3 & 0 & l_3 c_3 \\ s_3 & c_3 & 0 & l_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_3 c_{12} - s_3 s_{12} & -s_3 c_{12} - c_3 s_{12} & 0 & l_3 c_3 c_{12} - l_3 s_3 s_{12} + l_1 c_1 + l_2 c_{12} \\ c_3 s_{12} + s_3 c_{12} & -s_3 s_{12} + c_3 c_{12} & 0 & l_3 c_3 s_{12} + l_3 s_3 c_{12} + l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

forward kinematics
$$\begin{cases} x = l_3 c_3 c_{12} - l_3 s_3 s_{12} + l_1 c_1 + l_2 c_{12} \\ y = l_3 c_3 s_{12} + l_3 s_3 c_{12} + l_1 s_1 + l_2 s_{12} \end{cases}$$

$$J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \times (o_3 - o_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad O_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}; \quad O_2 = \begin{bmatrix} l_1 c_1 + \cancel{l_2 c_1 c_2 - s_1} \, l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}; \quad O_3 = \begin{bmatrix} l_3 c_3 c_{12} - l_3 s_3 s_{12} + l_1 c_1 + l_2 c_{12} \\ l_3 c_3 s_{12} + l_3 s_3 c_{12} + l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_3 c_3 s_{12} - l_3 s_3 c_{12} - l_1 s_1 - l_2 s_{12} & -l_3 c_3 s_{12} - l_3 s_3 c_{12} - l_2 s_{12} & -l_3 c_3 s_{12} - l_3 s_3 c_{12} \\ l_3 c_3 c_{12} - l_3 s_3 s_{12} + l_1 c_1 + l_2 c_{12} & l_3 c_3 c_{12} - l_3 s_3 s_{12} + l_2 c_{12} & l_3 c_3 c_{12} - l_3 s_3 s_{12} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\dot{x} = J \cdot \dot{\theta}$$

$$\ddot{x} = J \ddot{\theta} + \frac{dJ}{dt} \cdot \dot{\theta}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1(-l_3 c_3 s_{12} - l_3 s_3 c_{12} - l_1 s_1 - l_2 s_{12}) + \dot{\theta}_2(-l_3 c_3 s_{12} - l_3 s_3 c_{12} - l_2 s_{12}) + \dot{\theta}_3(-l_3 c_3 s_{12} - l_3 s_3 c_{12}) \\ \dot{\theta}_1(l_3 c_3 c_{12} - l_3 s_3 s_{12} + l_1 c_1 + l_2 c_{12}) + \dot{\theta}_2(l_3 c_3 c_{12} - l_3 s_3 s_{12} + l_2 c_{12}) + \dot{\theta}_3(l_3 c_3 c_{12} - l_3 s_3 s_{12}) \\ 0 \\ 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$$

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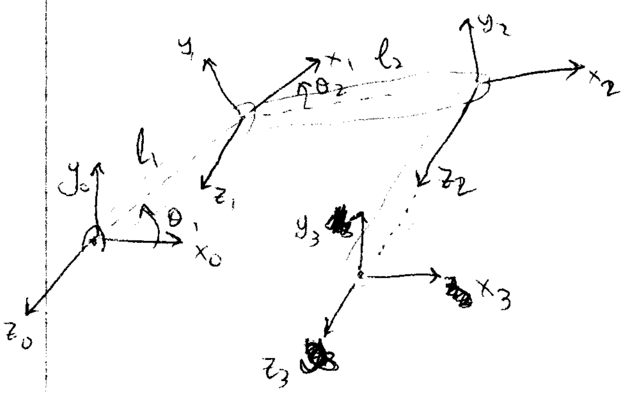
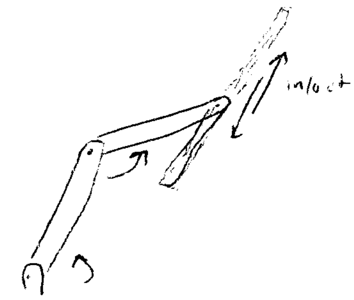
(3)





	a	d	α	θ
1	l_1	0	0	θ_1^*
2	l_2	0	0	θ_2^*
3	0	d_3^*	0	0

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{12} & c_{12} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{12} & c_{12} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

forward kinematics
$$\begin{cases} x = l_1 c_1 + l_2 c_{12} \\ y = l_1 s_1 + l_2 s_{12} \\ z = d_3 \end{cases}$$

$$J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

revolute   revolute   prismatic

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad o_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}, \quad o_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}, \quad o_3 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ d_3 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 \; \cancel{d_3} & \cancel{d_3} \; 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

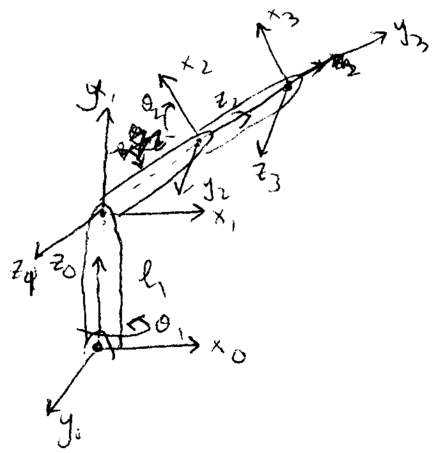
$$\dot{x} = J \dot{\theta}$$

$$\ddot{x} = J \ddot{\theta} + \frac{dJ}{dt} \cdot \dot{\theta}$$

Expand

Next

(4)



	a	d	α	θ
1	0	l_1	$90°$	θ_1^*
2	0	l_2	0	θ_2^*
3	0	d_2^*	$90°$	0

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = \begin{bmatrix} c_1c_2 & s_1 & c_1s_2 & s_1l_2 \\ c_2s_1 & -c_1 & s_1s_2 & -c_1l_2 \\ s_2 & 0 & -c_2 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} c_1c_2 & c_1s_2 & -s_1 & c_1s_2d_2 + s_1l_2 \\ c_2s_1 & s_1s_2 & c_1 & s_1s_2d_2 - c_1l_2 \\ s_2 & -c_2 & 0 & -c_2d_2 + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

forward kinematics
$$\begin{cases} x = c_1s_2d_2 + s_1l_2 \\ y = s_1s_2d_2 - c_1l_2 \\ z = -c_2d_2 + l_1 \end{cases}$$

$$z_2 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}, \quad z_1 = \begin{bmatrix} c_1s_2 \\ s_1s_2 \\ -c_2 \end{bmatrix}, \quad z_0 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} 0 \\ 0 \\ \ell_1 \end{bmatrix}; \quad O_2 = \begin{bmatrix} s_1 \ell_2 \\ -c_1 \ell_2 \\ \ell_1 \end{bmatrix}; \quad O_3 = \begin{bmatrix} c_1 s_2 d_2 + s_1 \ell_2 \\ s_1 s_2 d_2 - c_1 \ell_2 \\ -c_2 d_2 + \ell_1 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

revolute revolute Prismatic

$$z_0 \times (O_3 - O_0) = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} c_1 s_2 d_2 + s_1 \ell_2 \\ s_1 s_2 d_2 - c_1 \ell_2 \\ -c_2 d_2 + \ell_1 \end{bmatrix} = \begin{bmatrix} c_1 c_2 d_2 + c_1 \ell_1 \\ c_2 d_2 s_1 - s_1 \ell_1 \\ s_1^2 s_2 d_2 - s_1 c_1 \ell_2 + c_1^2 s_2 d_2 + c_1 s_1 \ell_2 \end{bmatrix} = \begin{bmatrix} c_1 c_2 d_2 + c_1 \ell_1 \\ c_2 d_2 s_1 - s_1 \ell_1 \\ (s_1^2 + c_1^2)(s_2 d_2) \end{bmatrix}$$

$$z_1 \times (O_3 - O_1) = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ -c_2 \end{bmatrix} \times \begin{bmatrix} c_1 s_2 d_2 + s_1 \ell_2 \\ s_1 s_2 d_2 - c_1 \ell_2 \\ -c_2 d_2 \end{bmatrix} = \begin{bmatrix} -s_1 s_2 c_2 d_2 + c_2 s_1 s_2 d_2 - c_1 c_2 \ell_2 \\ +c_1 c_2 d_2 s_2 - c_1 c_2 s_2 d_2 - s_1 c_2 \ell_2 \\ c_1 s_1 s_2^2 d_2 - c_1^2 s_2 \ell_2 - c_1 s_1 s_2^2 d_2 - s_1^2 s_2 \ell_2 \end{bmatrix} = \begin{bmatrix} -c_1 c_2 \ell_2 \\ -s_1 c_2 \ell_2 \\ (-c_1^2 - s_1^2) s_2 \ell_2 \end{bmatrix}$$

$$J = \begin{bmatrix} c_1 c_2 d_2 + c_1 \ell_1 & -c_1 c_2 \ell_2 & -s_1 \\ c_2 d_2 s_1 - s_1 \ell_1 & -s_1 c_2 \ell_1 & c_1 \\ (s_1^2 + c_1^2)(s_2 d_2) & (-c_1^2 - s_1^2) s_2 \ell_2 & 0 \\ s_1 & c_1 s_2 & 0 \\ -c_1 & s_1 s_2 & 0 \\ 0 & -c_2 & 0 \end{bmatrix}$$