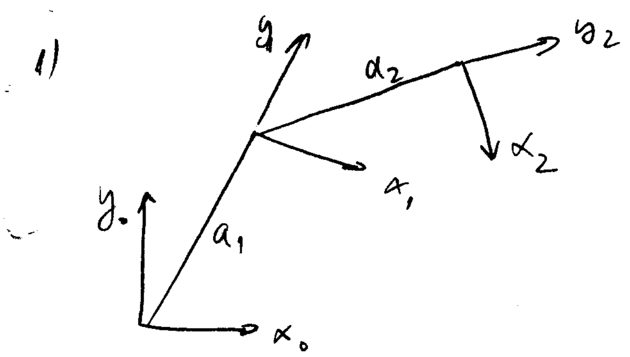
(1)

ANDREW ROSCA

Redo

−50

1)



	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_0^1 \qquad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_1 s_1 \\ s_2 & c_2 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{12} = T_0^2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_2 \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) Kinematics

$$x = a_1 c_1 + a_2 c_2 \qquad y = a_1 s_1 + a_2 s_2$$

$$R = \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) Velocities

$$J_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ a_1 c_1 + a_2 c_2 \\ -a_1 s_1 + a_2 s_2 \\ 0 \end{bmatrix} \qquad J_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix} \qquad J_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ a_1 c_1 + a_2 c_2 \\ a_1 s_1 + a_2 s_2 \\ 0 \end{bmatrix}$$

(2)

$$J_1 = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} \\ a_1 C_1 + a_2 C_{12} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad J_2 = \begin{bmatrix} -a_2 S_{12} \\ a_2 C_{12} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\cancel{J = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} \\ a_1 C_1 + a_2 C_{12} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}} \qquad \cancel{J = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} \\ a_1 C_1 + a_2 C_{12} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}$$

$$J = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} & -a_2 S_{12} \\ a_1 C_1 + a_2 C_{12} & a_2 C_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} V \\ \omega \end{bmatrix} = J \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

c) Inverse Kinematics

$$\begin{cases} a_1 C_1 + a_2 C_2 = x \\ a_1 S_1 + a_2 S_2 = y \end{cases}$$

d) Acceledation

$$\frac{d}{dt}(J) = \begin{bmatrix} -\cos\theta_1 \cdot a_1 - \cos(\theta_1+\theta_2) a_2 & -a_2 \cos(\theta_1+\theta_2) \\ -\sin(\theta_1) a_1 - \sin(\theta_1+\theta_2) a_2 & -\sin(\theta_1+\theta_2) a_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a_1 C_1 - a_2 C_{12} & -a_2 C_{12} \\ -S_1 a_1 - S_{12} a_2 & -S_{12} a_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = J' \qquad \ddot{x} = J \ddot{q} + J' \dot{q}$$

$$\frac{d\omega}{dt}$$

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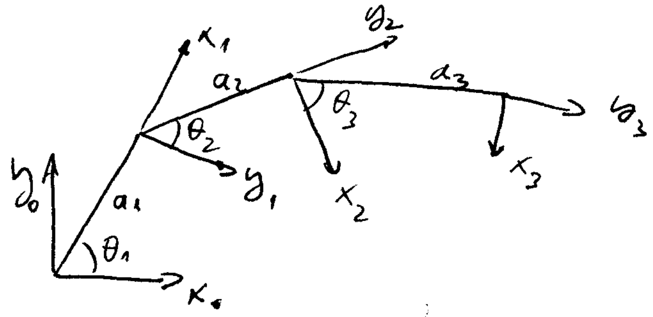
2. Same as previous step, except for the additional link.
We can build on the previous results



$$A_3 = \begin{bmatrix} C_3 & S_3 & 0 & C_3 a_3 \\ -S_3 & C_3 & 0 & S_3 a_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

$$T_0^3 = T_0^2 A_3 = \begin{bmatrix} C_{123} & S_{123} & 0 & C_3 a_3 + C_2 a_2 + a_1 C_1 \\ -S_{123} & C_{123} & 0 & S_3 a_3 + a_2 S_2 + a_1 S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) Kinematics

$$x = a_1 C_1 + a_2 C_2 + a_3 C_3$$
$$y = a_1 S_1 + a_2 S_2 + a_3 S_3$$

$$R = \begin{bmatrix} C_{123} & S_{123} & 0 \\ -S_{123} & C_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) Inverse Kinematics

$$\begin{cases} a_1 \cos\theta_1 + a_2 \cos\theta_2 + a_3 \cos\theta_3 = x \\ a_1 \sin\theta_1 + a_2 \sin\theta_2 + a_3 \sin\theta_3 = y \end{cases}$$

$$t = \sin\theta_3 \in [0,1]$$
$$p = \sqrt{1-t^2} = \cos\theta_3$$

$$\Rightarrow \cos\theta_1 = \frac{x - a_2\cos\theta_2 - a_3\cos\theta_3}{a_1} = \frac{x - a_2\cos\theta_2 - a_3 p}{a_1} = n$$

$$\Rightarrow \cancel{a_2\sqrt{1-n^2} = y} \Rightarrow \sin\theta_2 = \frac{y - a_1\sqrt{1-n^2} - a_3 t}{a_2}$$

(4)

$$\theta_2 = \arcsin\left(\frac{y - a_1\sqrt{1-n^2} - a_3 t}{a_2}\right) \qquad t \in [0,1] \qquad n = \frac{x - a_2\cos\theta_2 - a_3 \rho}{a_1}$$

$$\cancel{a_1 \sin\theta_2 + a_2\sqrt{1-\sin^2\theta_2}} =$$

c) Velocities

$$z_1 = z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad O_1 = \begin{bmatrix} a_1 c_1 \\ a_1 S_1 \\ 0 \end{bmatrix} \qquad O_2 = \begin{bmatrix} a_1 c_1 + a_2 c_2 \\ a_1 S_1 + a_2 S_2 \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} a_3 c_3 + a_2 c_2 + a_1 c_1 \\ a_3 S_3 + a_2 S_2 + a_1 S_1 \\ 0 \end{bmatrix} \qquad \cancel{z \times (O_2 - O) = -a_1 S_1 + a_1 c_1}$$

$$J_1 = \begin{bmatrix} z \times (O_3 - O_0) \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -a_3 S_3 - a_2 S_2 - a_1 S_1 \\ + a_3 c_3 + a_2 c_2 + a_1 c_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z \times \begin{bmatrix} a_3 c_3 + a_2 c_2 \\ a_3 S_3 + a_2 S_2 \\ 0 \end{bmatrix} \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -a_3 S_3 - a_2 S_2 \\ a_3 c_3 + a_2 c_2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} -a_3 S_3 \\ a_3 c_3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad J = \begin{bmatrix} -a_3 S_3 - a_2 S_2 - a_1 S_1 & -a_3 S_3 - a_2 S_2 & -a_3 S_3 \\ a_3 c_3 + a_2 c_2 + a_1 c_1 & a_3 c_3 + a_2 c_2 & a_3 c_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

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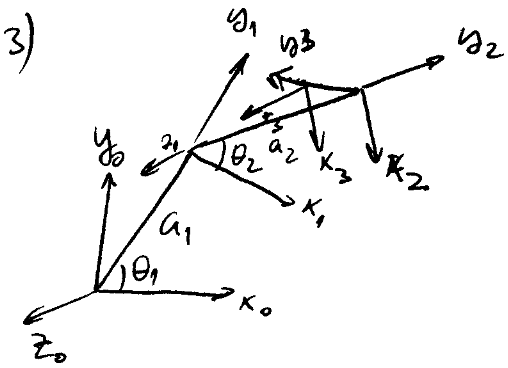
3)



	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	0	90	d_3^*	0

$$A_1 A_2 = T_0^2 = \begin{bmatrix} C_{12} & -S_{12} & 0 & a_1 C_1 + a_2 C_2 \\ S_{12} & C_{12} & 0 & a_1 S_1 + a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (see first problem)

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} C_{12} & 0 & S_{12} & a_1 C_1 + a_2 C_2 \\ S_{12} & 0 & -C_{12} & a_1 S_1 + a_2 S_2 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) Kinematics

$x = a_1 C_1 + a_2 C_2 \qquad y = a_1 S_1 + a_2 S_2 \qquad z = d_3$

$$R = \begin{bmatrix} C_{12} & 0 & S_{12} \\ S_{12} & 0 & -C_{12} \\ 0 & 1 & 0 \end{bmatrix}$$

b) Inverse Kinematics

$$\begin{cases} a_1 C_1 + a_2 C_2 = x \\ a_1 S_1 + a_2 S_2 = y \\ d_3 = z \end{cases}$$

c) Velocities

$$z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \cancel{z_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}} \quad z_3 = \begin{bmatrix} S_{12} \\ -C_{12} \\ 0 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad O_1 = \begin{bmatrix} a_1 C_1 \\ a_1 S_1 \\ 0 \end{bmatrix} \qquad O_2 = \begin{bmatrix} a_1 C_1 + a_2 C_2 \\ a_1 S_1 + a_2 S_2 \\ 0 \end{bmatrix} \qquad O_3 = \begin{bmatrix} a_1 C_1 + a_2 C_2 \\ a_1 S_1 + a_2 S_2 \\ d_3 \end{bmatrix}$$

$$z_1 \times (O_3 - O_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_1 C_1 + a_2 C_2 \\ a_1 S_1 + a_2 S_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} -a_1 S_1 - a_2 S_2 \\ a_1 C_1 + a_2 C_2 \\ 0 \end{bmatrix}$$

$$z_2 \times (O_3 - O_1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} +a_2 C_2 \\ a_2 S_2 \\ 0 \end{bmatrix} = \begin{bmatrix} -a_2 S_2 \\ a_2 C_2 \\ 0 \end{bmatrix}$$

$$z_3 \times (O_3 - O_2) = \begin{bmatrix} S_{12} \\ -C_{12} \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ d_3 \end{bmatrix} = \begin{bmatrix} -C_{12} d_3 \\ -S_{12} d_3 \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -a_1 S_1 - a_2 S_2 & -a_2 S_2 & -C_{12} d_3 \\ a_1 C_1 + a_2 C_2 & a_2 C_2 & -S_{12} d_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

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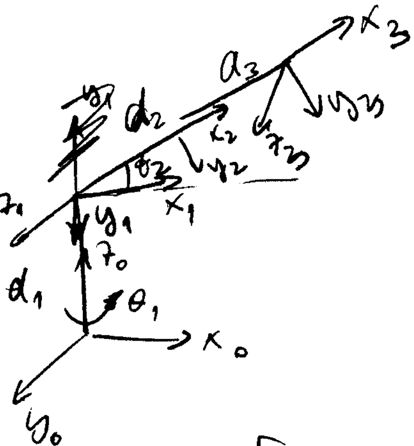
4)



	a_i	α_i	d_i	θ_i
~~1~~	~~0~~	~~0~~	~~d_1~~	~~θ_1~~
~~2~~				
~~3~~				

	a_i	α_i	d_i	θ_i
1	0	90	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3^*	0	0	~~0~~

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_0^1 \qquad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = T_0^2 = \begin{bmatrix} c_1 c_2 & -c_1 s_2 & s_1 & a_2 c_1 c_2 \\ s_1 c_2 & -s_1 s_2 & -c_1 & a_2 c_2 s_1 \\ s_2 & c_2 & 0 & a_2 s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_1 A_2 A_3 = T_0^3 = \begin{bmatrix} c_1 c_2 & -c_1 s_2 & s_1 & c_1 c_2 a_3 + a_2 c_2 c_1 \\ s_1 c_2 & -s_1 s_2 & -c_1 & s_1 c_2 a_3 + a_2 c_2 s_1 \\ s_2 & c_2 & 0 & s_2 a_3 + a_2 s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) Kinematics

$$x = a_2 c_2 c_1 + a_3 c_2 c_1 = c_1 c_2 (a_2 + a_3)$$

$$y = (a_2 + a_3) s_1 c_2$$

a_3 variable

$$z = a_3 s_2 + a_2 s_2 + d_1 = d_1 + s_2 (a_2 + a_3)$$

$$R = \begin{bmatrix} c_1 c_2 & -c_1 s_2 & s_1 \\ s_1 c_2 & -s_1 s_2 & -c_1 \\ s_2 & c_2 & 0 \end{bmatrix}$$

b) inverse Kinematics

$$\begin{cases} c_1 c_2 (a_2+a_3) = x \\ s_1 c_2 (a_2+a_3) = y \\ d_1 + s_2 (a_2+a_3) = z \end{cases} \Rightarrow \cancel{c_2 =} \quad c_2 = \frac{x}{c_1(a_2+a_3)}$$

$$\cancel{s_1 = \frac{y}{\frac{x}{c_1(a_2+a_3)}(a_2+a_3)} \cdot s_1 = \frac{y c_1}{x s_1} = \frac{y\sqrt{1-s_1^2}}{x s_1}}$$

$$c_2 = \frac{x}{c_1(a_2+a_3)}$$

$$\Rightarrow s_1 = \frac{y}{\cancel{(a_2+a_3)}\frac{x}{c_1\cancel{(a_2+a_3)}}} = \frac{c_1 y}{x}$$

$$s_1 = \frac{\sqrt{1-s_1^2}\, y}{x} \qquad s_1^2 = \frac{(1-s_1^2) y^2}{x^2}$$

$$s_1^2(x^2+1) = 1+y^2 \Rightarrow s_1 = \pm\sqrt{\frac{1+y^2}{1+x^2}}$$

$$\theta_1^1 = \arcsin\sqrt{\frac{y^2+1}{x^2+1}} \qquad \theta_1^2 = -\arcsin\sqrt{\frac{y^2+1}{x^2+1}}$$

Velocities

$$z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \quad z_2 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} = z_3 \quad o_3 - o_0 = \begin{bmatrix} a_3 c_2 c_1 + a_2 c_2 c_1 \\ c_2 s_1 (a_2+a_3) \\ s_2(a_2+a_3) + d_1 \end{bmatrix}$$

$$o_3 - o_1 = \begin{bmatrix} c_2 c_1 (a_2+a_3) \\ c_2 s_1 (a_2+a_3) \\ s_2 (a_2+a_3) \end{bmatrix} \qquad o_3 - o_2 = \begin{bmatrix} a_3 c_1 c_2 \\ a_3 s_1 c_2 \\ a_3 s_2 \end{bmatrix}$$

$$z_1 \times (o_3 - o_0) = \begin{bmatrix} -c_1[s_2(a_2+a_3) + d_1] \\ 0 \\ 0 \end{bmatrix}$$

$\cancel{z_2 \times (O_3 - O_1) = \begin{bmatrix} 0 \\ s_1 s_2 (a_2 + a_3 \end{bmatrix}}$

$$z_2 \times (O_3 - O_1) = \begin{bmatrix} -c_1 s_2 (a_2 + a_3) \\ -s_1 s_2 (a_2 + a_3) \\ c_2 s_1^2 (a_2 + a_3) + c_1^2 c_2 (a_2 + a_3) \end{bmatrix}$$

$$= \begin{bmatrix} -c_1 s_2 (a_2 + a_3) \\ -s_1 s_2 (a_2 + a_3) \\ c_2 (a_2 + a_3) \end{bmatrix} \qquad z_3 \times (O_3 - O_2) = \Big[$$