Software package for a RRR:RRR robot with known D-H table

1. Forward Kinematics

Inputs: $\theta_1, \theta_2, \ldots, \theta_6$
Outputs: $x, y, z, \omega_1, \omega_2, \omega_3$

- The x, y, and z values you can get from the matrix multiplication as the first three entries in the fourth column.
- The $\omega_1, \omega_2, \omega_3$ values you can get from the rotational part of the multiplied matrices.

2. Inverse Kinematics

Inputs: $x, y, z, \omega_1, \omega_2, \omega_3$
Outputs: $\theta_1, \theta_2, \ldots, \theta_6$

- Use the D-H table to construct the symbolically the homogenous transformation matrix
- Solve the 12 equations w/ 6 unknowns.

3. Velocity Kinematics

Inputs: $\theta_1, \theta_2, \ldots, \theta_6, \dot{\theta}_1, \dot{\theta}_2, \ldots, \dot{\theta}_6$
Outputs: $v_x, v_y, v_z, \omega_x, \omega_y, \omega_z$

- Construct the Jacobian as follows:

$$J = [J_1 J_2 \ldots J_n]$$

, where if joint *(i)* is revolute

$$J_i = \begin{bmatrix} Z_{i-1} \times (O_n - O_{i-1}) \\ Z_{i-1} \end{bmatrix}$$

, and if joint *(i)* is prismatic

$$J_i = \begin{bmatrix} Z_{i-1} \\ 0 \end{bmatrix}$$

Where Z_i is the first three elements in the 3rd column of the T_0^i matrix, and O_i is the first three elements in the 4th column of the T_0^i matrix.

4. Inverse Velocity Kinematics

Inputs: θ_1, θ_2, … , θ_6, v_x, v_y, v_z, ω_x, ω_y, ω_z
Outputs: $\dot{\theta}_1$, $\dot{\theta}_2$, … , $\dot{\theta}_6$
- Get symbolically the inverse Jacobian matrix $J^{-1}(q)$ and compute the following:

$$\dot{q} = J^{-1}(q)\dot{X}$$

5. Acceleration Kinematics

Inputs: θ_1, θ_2, … , θ_6, $\dot{\theta}_1$, $\dot{\theta}_2$, … , $\dot{\theta}_6$, $\ddot{\theta}_1$, $\ddot{\theta}_2$, … , $\ddot{\theta}_6$
Outputs: a_x, a_y, a_z, $\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$

- Get symbolically $\frac{d}{dt}J(q)$ as well as $\dot{q}$ and compute the following:

$$\ddot{X} = J(q)\ddot{q} + \left(\frac{d}{dt}J(q)\right)\dot{q}$$

6. Inverse Acceleration Kinematics

Inputs: θ_1, θ_2, … , θ_6, $\dot{\theta}_1$, $\dot{\theta}_2$, … , $\dot{\theta}_6$, a_x, a_y, a_z, $\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$
Outputs: $\ddot{\theta}_1$, $\ddot{\theta}_2$, … , $\ddot{\theta}_6$

$$\ddot{q} = J^{-1}(q)\ddot{X} - J^{-1}(q)\frac{d}{dt}J(q)\dot{q}$$