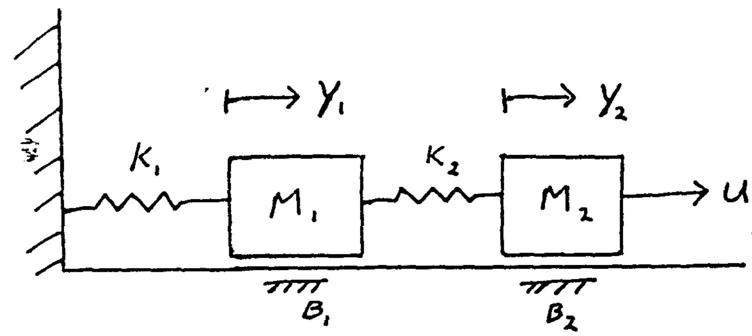
Sample Exam No. 2

1 Consider the coupled spring–mass system below.



Mass M_1 is connected by a spring to a rigid wall. Mass M_2 is coupled to mass M_1 by a second spring. Both masses are sli[illegible] with frictio[illegible] B_1 [illegible] B_2 represent the coefficients of viscous friction [illegible] [illegible]plied to M_2 as shown. Let y_1, y_2 repre[illegible] [illegible] of the two ma[illegible].

a) Write the total kinetic energy of the system.

b) Write the potential energy of the system.

c) Form the Lagrangian and derive the equations of motion for the system using the Euler–Lagrange equations.

2 Consider the nonlinear system shown below.

$$\ddot{\theta}_1 + (3\sin\theta_1)\ddot{\theta}_2 + \dot{\theta}_1\dot{\theta}_2 + \sin\theta_2 = \tau_1 + $$
$$\ddot{\theta}_2 - (2\sin\theta_1)\ddot{\theta}_1 - \dot{\theta}_1\dot{\theta}_2 = $$

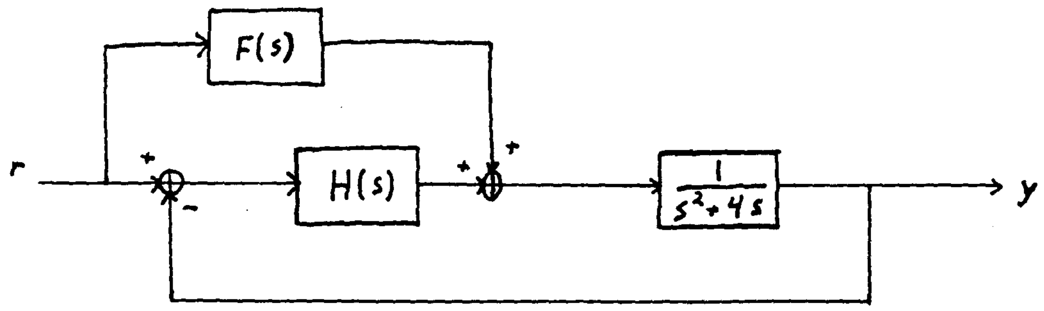
a) Find an inverse dynamics control [illegible] (τ_1, τ_2) to transform this system into the double integrator system

$$\ddot{\theta}_1 = a_1$$
$$\ddot{\theta}_2 = a_2$$

where a_1, a_2 are new (outer loop) controls to be designed.

b) Design the outer loop controls (a_1, a_2) so that the closed loop system is decoupled, with natural frequency $\omega = 10$ and damping ratio $\zeta = 1$ for both degrees of freedom.

3 Consider the block diagram below.



Design a PD compensator $H(s)$ and a feedforward compensator $F(s)$ so that the closed loop system

a) is critically damped with closed loop natural frequency $\omega = 12$.

b) tracks a desired reference trajectory $\theta^d(t) = \cos 3t$.

4 Consider the system of differential equations

$\dot{x}_1 = x_2$

$\dot{x}_2 = -x_1 - x_1^2 x_2$

a) Show that $(0,0)$ is an equilibrium point of the system.

b) Using the Lyapunov function candidate

$V = x_1^2 + x_2^2$

show that the equilibrium solution is asymptotically stable.