180/100

Excellent Job on assignment A!

Robotics CS460

Homework

Some of chap 1 + text problems (numeric)

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1) RRR manipulator configuration in three-space

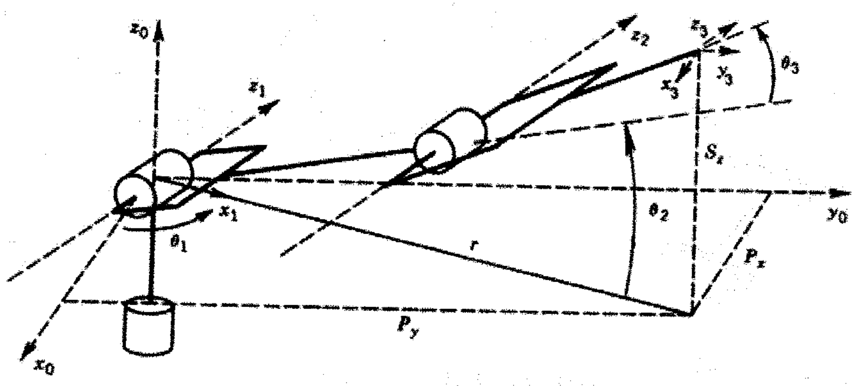

Fig. 1 Simplified drawing of the RRR manipulator with transformation frames appropriately marked.

$$\theta_1 = A\tan\left(\frac{P_y}{P_x}\right)$$ **Equation 1**

Further, to find angles θ_2 and θ_3 for the elbow manipulator, given θ_1, we consider the plane formed by the second and third links as shown in Figure 2.

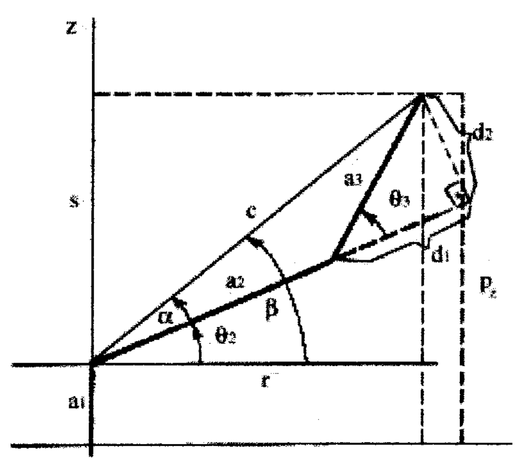

Fig. 2 Projection onto the plane formed by links two and three

Since the motion of links two and three is planar, we can obtain the value for θ_3 with the application of the Law of Cosines:

$$Cos(\alpha) = \frac{a_2^2 + c^2 - a_3^2}{2 \cdot a_2 \cdot c}$$, by Law of Cosines

$$Cos(\alpha) = \frac{a_2 + d_1}{c}$$

$$\therefore d_1 = \frac{\left(a_2^2 + c^2 - a_3^2\right) \cdot c}{2 \cdot a_2 \cdot c} - a_2 = \frac{a_2^2 + c^2 - a_3^2 - 2 \cdot a_2^2}{2 \cdot a_2} = \frac{c^2 - a_2^2 - a_2^2}{2 \cdot a_2}$$

$$Cos(\theta_3) = \frac{d_1}{a_3} = \frac{\dfrac{c^2 - a_2^2 - a_3^2}{2 \cdot a_2}}{a_3} = \frac{c^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3} = D$$

$$Sin(\theta_3) = \pm\sqrt{1 - D^2}$$

$$Tan(\theta_3) = \frac{Sin(\theta_3)}{Cos(\theta_3)} = \frac{\pm\sqrt{1 - D^2}}{D} = \frac{\pm\sqrt{1 - \left(\dfrac{c^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}\right)^2}}{\dfrac{c^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}}$$

, therefore θ_3 is given by:

$$\theta_3 = A\tan\left(\frac{\pm\sqrt{1 - \left(\dfrac{c^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}\right)^2}}{\dfrac{c^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}}\right) =$$

$$= A\tan\left(\frac{\pm\sqrt{1 - \left(\dfrac{r^2 + s_z^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}\right)^2}}{\dfrac{r^2 + s_z^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}}\right) =$$

$$= A\tan\left(\frac{\pm\sqrt{1 - \left(\dfrac{p_x^2 + p_y^2 + (p_z - a_1)^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}\right)^2}}{\dfrac{p_x^2 + p_y^2 + (p_z - a_1)^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}}\right)$$

Equation 2

Similarly θ_2 is given as:

$$\theta_2 = \beta - \alpha$$

$$Tan(\beta) = \frac{s}{r} \Rightarrow \beta = A\tan\left(\frac{p_z - a_1}{\sqrt{p_x^2 + p_y^2}}\right)$$

$$Tan(\alpha) = \frac{d_2}{a_2 + d_1}$$

$$d_2 = a_3 \cdot Sin(\theta_3)$$

$$d_1 = a_3 \cdot Cos(\theta_3)$$

$$\therefore Tan(\alpha) = \frac{a_3 \cdot Sin(\theta_3)}{a_2 + a_3 \cdot Cos(\theta_3)} \Rightarrow \alpha = A\tan\left(\frac{a_3 \cdot Sin(\theta_3)}{a_2 + a_3 \cdot Cos(\theta_3)}\right)$$

, therefore θ_2 is given by:

$$\theta_2 = A\tan\left(\frac{p_z - a_1}{\sqrt{p_x^2 + p_y^2}}\right) - A\tan\left(\frac{a_3 \cdot Sin(\theta_3)}{a_2 + a_3 \cdot Cos(\theta_3)}\right)$$

Equation 3

Make a note that the square root from the equation of θ_3 has two possible solutions that correspond to the elbow-up position, and elbow-down position, respectively (Fig. 3).

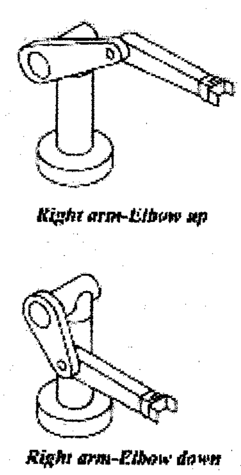

Fig. 3 Right-arm configured RRR manipulator in the elbow-up and elbow-down positions

Also, in the configuration as this one, we did not include the offset of links two and three in respect to z_0 axis, thus when $p_x = p_y = 0$, there are infinitely many solutions for θ_1 (no left/right-arm configurations).

2) RRP manipulator configuration in two-space

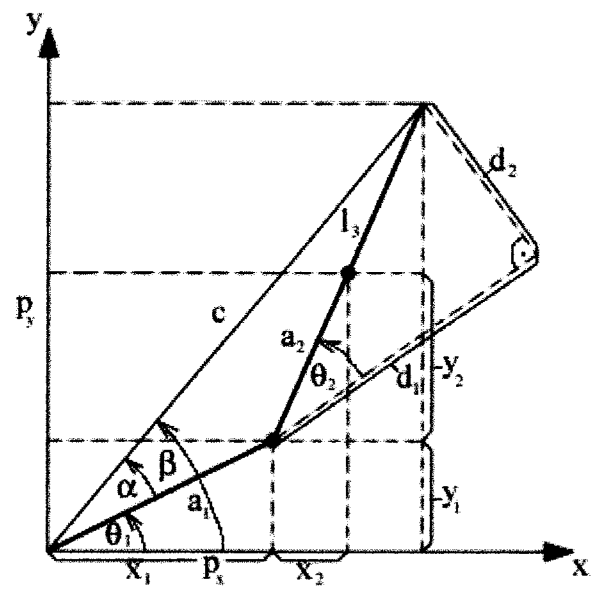

Fig. 2 Projection onto the plane formed by all three links

Since the motion of links two and three is planar, we can obtain the value for θ_3 with the application of the Law of Cosines:

$$Cos(\alpha) = \frac{a_1^2 + c^2 - (a_2 + l_3)^2}{2 \cdot a_1 \cdot c}$$, by Law of Cosines

$$Cos(\alpha) = \frac{a_1 + d_1}{c}$$

$$\therefore d_1 = \frac{\left(a_1^2 + c^2 - (a_2 + l_3)^2\right) \cdot c}{2 \cdot a_1 \cdot c} - a_1 = \frac{a_1^2 + c^2 - (a_2 + l_3)^2 - 2 \cdot a_1^2}{2 \cdot a_1} = \frac{c^2 - a_1^2 - (a_2 + l_3)^2}{2 \cdot a_1}$$

$$Cos(\theta_2) = \frac{d_1}{a_2 + l_3} = \frac{\frac{c^2 - a_1^2 - (a_2 + l_3)^2}{2 \cdot a_1}}{a_2 + l_3} = \frac{c^2 - a_1^2 - (a_2 + l_3)^2}{2 \cdot a_1 \cdot (a_2 + l_3)} = D$$

$$Sin(\theta_3) = \pm\sqrt{1 - D^2}$$

$$Tan(\theta_2) = \frac{Sin(\theta_2)}{Cos(\theta_2)} = \frac{\pm\sqrt{1-D^2}}{D} = \frac{\pm\sqrt{1-\left(\frac{c^2-a_1^2-(a_2+l_3)^2}{2\cdot a_1\cdot(a_2+l_3)}\right)^2}}{\frac{c^2-a_1^2-(a_2+l_3)^2}{2\cdot a_1\cdot(a_2+l_3)}}$$

, therefore θ_2 is given by:

$$\theta_2 = A\tan\left(\frac{\pm\sqrt{1-\left(\frac{c^2-a_1^2-(a_2+l_3)^2}{2\cdot a_1\cdot(a_2+l_3)}\right)^2}}{\frac{c^2-a_1^2-(a_2+l_3)^2}{2\cdot a_1\cdot(a_2+l_3)}}\right) =$$

$$= A\tan\left(\frac{\pm\sqrt{1-\left(\frac{p_x^2+p_y^2-a_1^2-(a_2+l_3)^2}{2\cdot a_1\cdot(a_2+l_3)}\right)^2}}{\frac{p_x^2+p_y^2-a_1^2-(a_2+l_3)^2}{2\cdot a_1\cdot(a_2+l_3)}}\right)$$

Equation 1

Similarly θ_1 is given as:

$$\theta_1 = \beta - \alpha$$

$$Tan(\beta) = \frac{p_y}{p_x} \Rightarrow \beta = A\tan\left(\frac{p_y}{p_x}\right)$$

$$Tan(\alpha) = \frac{d_2}{a_1+d_1}$$

$$d_2 = (a_2+l_3)\cdot Sin(\theta_2)$$

$$d_1 = (a_2+l_3)\cdot Cos(\theta_2)$$

$$\therefore Tan(\alpha) = \frac{(a_2+l_3)\cdot Sin(\theta_2)}{a_1+(a_2+l_3)\cdot Cos(\theta_2)} \Rightarrow \alpha = A\tan\left(\frac{(a_2+l_3)\cdot Sin(\theta_2)}{a_1+(a_2+l_3)\cdot Cos(\theta_2)}\right)$$

, therefore θ_1 is given by:

$$\theta_1 = A\tan\left(\frac{p_y}{p_x}\right) - A\tan\left(\frac{(a_2+l_3)\cdot Sin(\theta_2)}{a_1+(a_2+l_3)\cdot Cos(\theta_2)}\right)$$

Equation 2

For the prismatic joint we have the following formula:

$$l_3 = \sqrt{(p_y - y_1 - y_2)^2 + (p_x - x_1 - x_2)^2}$$
$$l_3 = \sqrt{(p_y - a_1 \cdot Sin(\theta_1) - a_2 \cdot Sin(\theta_2))^2 + (p_x - a_1 \cdot Cos(\theta_1) - a_2 \cdot Cos(\theta_2))^2}$$

Equation 3

1-14)

$$Cos(\theta) = \frac{l^2 + l^2 - d^2}{2 \cdot l \cdot l} = \frac{2 \cdot l^2 - d^2}{2 \cdot l^2} = 1 - \frac{d^2}{2 \cdot l^2}$$
$$\frac{d^2}{2 \cdot l^2} = 1 - Cos(\theta) \Rightarrow d^2 = 2 \cdot l^2 \cdot (1 - Cos(\theta)) \Rightarrow d = \sqrt{2} \cdot l \cdot \sqrt{1 - Cos(\theta)} \backslash$$
$$\therefore d = l \cdot \sqrt{2 \cdot (1 - Cos(\theta))}$$

10bit accuracy
$l = 1m$
$\theta = 90°$

linear link resolution:

$$resolution = \frac{l \cdot \sqrt{2 \cdot (1 - Cos(\theta))}}{2^n} = \frac{1 \cdot \sqrt{2}}{2^{10}} = 1.381 \cdot 10^{-3} m$$

rotational link resolution:

$$resolution = \frac{l \cdot \pi \cdot \theta}{2^n \cdot 180°} = \frac{1 \cdot 3.14159 \cdot 90°}{2^{10} \cdot 180°} = 1.5339 \cdot 10^{-3} m$$

1-15)

$l = 50cm = 0.5m$
$\theta = 180°$
8-bit encoder

$$resolution = \frac{l \cdot \pi \cdot \theta}{2^n \cdot 180°} = \frac{0.5 \cdot 3.14159 \cdot 180°}{2^8 \cdot 180°} = 1.5708 \cdot 10^{-3} m$$

1-16)

l = 50cm = 0.5m
$\theta = 180°$
8-bit encoder
50:1 gear ratio

$$resolution = \frac{l \cdot \pi \cdot \theta}{2^n \cdot 180° \cdot 50} = \frac{0.5 \cdot 3.14159 \cdot 180°}{2^8 \cdot 180° \cdot 50} = 1.2272 \cdot 10^{-4}\, m$$

1-19)

Note: Refer to Problem 2 Figure 2 without the additional link three.

$$\theta_1 = \beta - \alpha$$

$$Tan(\beta) = \frac{p_y}{p_x} \Rightarrow \beta = A\tan\left(\frac{p_y}{p_x}\right)$$

$$Tan(\alpha) = \frac{d_2}{a_1 + d_1}$$

$$d_2 = a_2 \cdot Sin(\theta_2)$$

$$d_1 = a_2 \cdot Cos(\theta_2)$$

$$\therefore Tan(\alpha) = \frac{a_2 \cdot Sin(\theta_2)}{a_1 + a_2 \cdot Cos(\theta_2)} \Rightarrow \alpha = A\tan\left(\frac{a_2 \cdot Sin(\theta_2)}{a_1 + a_2 \cdot Cos(\theta_2)}\right)$$

, therefore θ_1 is given by:

$$\theta_1 = A\tan\left(\frac{p_y}{p_x}\right) - A\tan\left(\frac{a_2 \cdot Sin(\theta_2)}{a_1 + a_2 \cdot Cos(\theta_2)}\right)$$

1-20)

$$a_1 = a_2 = 1$$

$$\theta_1 = \frac{\pi}{6} = 30°$$

$$\theta_2 = \frac{\pi}{2} = 90°$$

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x,y = ?

$$x = a_1 \cdot Cos(\theta_1) + a_2 \cdot Cos(\theta_1 + \theta_2) = 0.366m$$

$$y = a_1 \cdot Sin(\theta_1) + a_2 \cdot Sin(\theta_1 + \theta_2) = 1.366m$$

1-21)

$a_1 = a_2 = 1$
$x = y = 0.5$

$\theta_1, \theta_2 = ?$

$$\theta_2 = A\tan\left(\frac{\pm\sqrt{1-\left(\frac{x^2+y^2-a_1^2-a_2^2}{2\cdot a_1\cdot a_2}\right)^2}}{\frac{x^2+y^2-a_1^2-a_2^2}{2\cdot a_1\cdot a_2}}\right) =$$

$$= A\tan\left(\frac{\pm\sqrt{1-\left(\frac{0.5^2+0.5^2-1^2-1^2}{2\cdot 1\cdot 1}\right)^2}}{\frac{0.5^2+0.5^2-1^2-1^2}{2\cdot 1\cdot 1}}\right) = \pm 59.036°$$

$$\theta_{1.1} = A\tan\left(\frac{y}{x}\right) - A\tan\left(\frac{a_2\cdot Sin(\theta_2)}{a_1 + a_2\cdot Cos(\theta_2)}\right) =$$

$$= A\tan\left(\frac{0.5}{0.5}\right) - A\tan\left(\frac{1\cdot Sin(59.036°)}{1+1\cdot Cos(59.036°)}\right) = 15.482°$$

$$\theta_{1.2} = A\tan\left(\frac{y}{x}\right) - A\tan\left(\frac{a_2\cdot Sin(\theta_2)}{a_1 + a_2\cdot Cos(\theta_2)}\right) =$$

$$= A\tan\left(\frac{0.5}{0.5}\right) - A\tan\left(\frac{1\cdot Sin(-59.036°)}{1+1\cdot Cos(-59.036°)}\right) = 74.518°$$