CpE360

Excellent
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Homework #5

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Robotics Homework #5

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<< Calculus`VectorAnalysis`

Question 5.1

We start off with :

$z_0 = \{\{0,\ 0,\ 1\}\}^T;$
$z_1 = \{\{0,\ 0,\ 1\}\}^T;$
$O_0 = \{\{0,\ 0,\ 0\}\}^T;$

$O_c = \{\{a_1\, c_1 + a_c\, c_{12},\ a_1\, s_1 + a_c\, s_{12},\ 0\}\};$

$O_1 = \{\{a_1\, c_1,\ a_1\, s_1,\ 0\}\};$

$z_0\ (O_c - O_0)$

$$\begin{aligned}
&\{\{\{\{0,\ 0,\ 1\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}} \\
&\quad (-\{\{0,\ 0,\ 0\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}} + a_1\, c_1 + a_c\, c_{12}), \\
&\ \{\{0,\ 0,\ 1\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}} \\
&\quad (-\{\{0,\ 0,\ 0\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}} + a_1\, s_1 + a_c\, s_{12}), \\
&\ -\{\{0,\ 0,\ 0\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}} \\
&\quad \{\{0,\ 0,\ 1\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}}\}\}
\end{aligned}$$

$z_1\ (O_c - O_1)$

$\{\{\{\{0, 0, 1\}\}^{\{\{\text{Cos}[\theta_1]\ \text{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}}\ a_c\, c_{12},$
$\{\{0, 0, 1\}\}^{\{\{\text{Cos}[\theta_1]\ \text{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}}\ a_c\, s_{12},\ 0\}\}$

So the Jacobian Matrix is :

$$J = \begin{pmatrix} -a_1 s_1 - a_1 s_{12} & a_c s_{12} & 0 \\ a_1 c_1 + a_c c_{12} & a_c c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Question 5.2

The first Jacobian term is :

$O_0 = \{\{0, 0, 0\}\};$
$O_1 = \{\{0, 0, 0\}\};$

$$O_2 = \begin{pmatrix} a_2 c_1 c_2 \\ a_2 s_1 c_2 \\ a_2 s_2 \end{pmatrix};$$

$$O_3 = \begin{pmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} \\ a_2 s_1 c_2 + a_3 s_1 c_{23} \\ a_2 s_2 + a_3 s_{23} \end{pmatrix};$$

$$z_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix};$$

$$z_1 = \begin{pmatrix} s_1 \\ -c_1 \\ 0 \end{pmatrix};$$

$$z_2 = \begin{pmatrix} s_1 \\ -c_1 \\ 0 \end{pmatrix};$$

$z_0\ (O_3 - O_0)$

$\{\{0\}, \{0\}, \{\{\{0, 0, 0\}\} + \{\{a_2 c_1 c_2 + a_3 c_1 c_{23}\}, \{a_2 c_2 s_1 + a_3 c_{23} s_1\}, \{a_2 s_2 + a_3 s_{23}\}\}\}\}$

$z_1\ (O_3 - O_1)$

$\{\{(\{\{0, 0, 0\}\} + \{\{a_2 c_1 c_2 + a_3 c_1 c_{23}\}, \{a_2 c_2 s_1 + a_3 c_{23} s_1\}, \{a_2 s_2 + a_3 s_{23}\}\}) s_1\},$
$\{-(\{\{0, 0, 0\}\} + \{\{a_2 c_1 c_2 + a_3 c_1 c_{23}\}, \{a_2 c_2 s_1 + a_3 c_{23} s_1\}, \{a_2 s_2 + a_3 s_{23}\}\}) c_1\}, \{0\}\}$

$z_2\ (O_3 - O_2)$

$\{\{a_3 c_1 c_{23} s_1\}, \{-a_3 c_1 c_{23} s_1\}, \{0\}\}$

$J_{11} = \{\{ z_0\ (O_3 - O_0),\ z_1\ (O_3 - O_1),\ z_2\ (O_3 - O_2)\ \}\}$

Thus

$$J_{11} = \begin{pmatrix} -a_2 s_1 c_2 - a_3 s_1 c_{23} & -a_2 s_2 c_1 - a_3 s_{23} c_1 & -a_3 c_1 s_{23} \\ a_2 c_1 c_2 + a_3 c_1 c_{23} & a_2 s_1 s_2 + a_3 s_1 s_{23} & -a_3 s_1 s_{23} \\ 0 & a_2 c_2 + a_3 c_{23} & a_3 c_{23} \end{pmatrix}$$

$\{\{-a_2 c_2 s_1 - a_3 c_{23} s_1, -a_2 c_1 s_2 - a_3 c_1 s_{23}, -a_3 c_1 s_{23}\},$
$\{a_2 c_1 c_2 + a_3 c_1 c_{23}, a_2 s_1 s_2 + a_3 s_1 s_{23}, -a_3 s_1 s_{23}\}, \{0, a_2 c_2 + a_3 c_{23}, a_3 c_{23}\}\}$

Det[%]

$a_2^2 a_3 c_1^2 c_2 c_{23} s_2 + a_2 a_3^2 c_1^2 c_{23}^2 s_2 - a_2^2 a_3 c_2 c_{23} s_1^2 s_2 - a_2 a_3^2 c_{23}^2 s_1^2 s_2 - a_2^2 a_3 c_1^2 c_2^2 s_{23} -$
$a_2 a_3^2 c_1^2 c_2 c_{23} s_{23} - a_2^2 a_3 c_2^2 s_1^2 s_{23} - 3 a_2 a_3^2 c_2 c_{23} s_1^2 s_{23} - 2 a_3^3 c_{23}^2 s_1^2 s_{23}$

Question 5.3

$O_0 = \{\{0, 0, 0\}\}^T;$
$O_1 = \{\{0, 0, d_1\}\}^T;$
$O_2 = \{\{0, 0, d_2\}\}^T;$
$O_3 = \{\{d_2 s_2 c_1, d_2 s_2 s_1, d_1 + d_2 c_2\}\}^T;$

$z_0 = \{\{0, 0, 1\}\};$
$z_1 = \{\{s_1, c_1, 0\}\};$
$z_2 = \{\{s_2 c_1, s_2 s_1, c_2\}\};$

$z_0\ (O_3 - O_0)$

$\{\{0, 0, -\{\{0, 0, 0\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}} +$
$\{\{c_1 d_2 s_2, d_2 s_1 s_2, d_1 + c_2 d_2\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}}\}\}$

$z_1\ (O_3 - O_1)$

$\{\{(-\{\{0, 0, d_1\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}} +$
$\{\{c_1 d_2 s_2, d_2 s_1 s_2, d_1 + c_2 d_2\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}}) s_1,$
$(-\{\{0, 0, d_1\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}} +$
$\{\{c_1 d_2 s_2, d_2 s_1 s_2, d_1 + c_2 d_2\}\}^{\{\{\mathrm{Cos}[\theta_1]\ \mathrm{Cos}[\theta_3],0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,1\}\}}) c_1,$
$0\}\}$

$z_2 \ (O_3 - O_2)$

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{{(-{{0, 0, d2}}^{{Cos[θ1] Cos[θ3],0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,1}} +
    {{c1 d2 s2, d2 s1 s2, d1 + c2 d2}}^{{Cos[θ1] Cos[θ3],0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,1}}) c1 s2,
  (-{{0, 0, d2}}^{{Cos[θ1] Cos[θ3],0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,1}} +
    {{c1 d2 s2, d2 s1 s2, d1 + c2 d2}}^{{Cos[θ1] Cos[θ3],0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,1}}) s1 s2,
  (-{{0, 0, d2}}^{{Cos[θ1] Cos[θ3],0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,1}} +
    {{c1 d2 s2, d2 s1 s2, d1 + c2 d2}}^{{Cos[θ1] Cos[θ3],0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,1}})
  c2}}
```

$$J = \begin{pmatrix} s_1 s_2 d_2 & -c_1 c_2 d_2 & s_2 c_1 \\ c_1 s_2 d_2 & s_1 c_2 d_2 & s_2 s_1 \\ 0 & -s_2 d_2 & c_2 \\ 0 & s_1 & 0 \\ 0 & c_1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Question 5.4

We have that

$\alpha_1 \alpha_4 - \alpha_2 \alpha_3 = 0$

$\alpha_1 \alpha_4 - \alpha_2 \alpha_3 = (-a_1 s_1 - a_2 s_{12}) (a_1 c_{12}) + (a_1 s_{12}) (a_1 c_1 + a_2 c_{12})$

Which is equivalent to

$a_1^2 s_2$

Question 5.5

$O_0 = \{\{0, 0, 0\}\}^T;$

$O_3 = \{\{d_3 s_1, d_3 c_1, 0\}\}^T;$

$z_0 = \{\{0, 0, 1\}\}^T;$

$z_1 = \{\{0, 0, 1\}\}^T;$

$z_2 = \{\{-s_1, c_1, 0\}\}^T;$

$z_0 * (O_3 - O_0)$

```
{{0, 0, -{{0, 0, 0}}^{{Cos[θ1] Cos[θ3],0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,1}} +
    {{c1 d2 s2, d2 s1 s2, d1 + c2 d2}}^{{Cos[θ1] Cos[θ3],0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,1}}}}
```

$$J = \begin{pmatrix} z_0 * (O_3 - O_0) & z_1 & z_2 \\ z_0 & 0 & 0 \end{pmatrix}$$

$$J = \begin{pmatrix} c_1\, d_3 & 0 & -s_1 \\ s_1\, d_3 & 0 & c_1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

{{$c_1\, d_3$, 0, $-s_1$}, {$d_3\, s_1$, 0, c_1}, {0, 1, 0}, {0, 0, 0}, {0, 0, 0}, {1, 0, 0}}

$$\text{Det}\left[\begin{pmatrix} c_1\, d_3 & 0 & -s_1 \\ s_1\, d_3 & 0 & c_1 \\ 0 & 1 & 0 \end{pmatrix}\right]$$

$-c_1^2\, d_3 - d_3\, s_1^2$

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Question 5.6

We know that for a cartesian manipulator all joints will be prismatic and so

$$J = \begin{pmatrix} z_0 & z_1 & z_2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$