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# ROBOTCS

## Home-Work - 5

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Home-work 5

5.1 For the three-link planar manipulator of example 5.1.1 compute the vector $O_c$ and derive the Jacobian (5.1.29)

For three link planar manipulator,

let $a_c$ = distance from Joint to $O_c$ and $a_e$ = length of link 1.

Then $O_c = (x_c, y_c, z_c)^T$ where

$$x_c = a_1 C_1 + a_c C_{12}$$
$$y_c = a_1 S_1 + a_c C_{12}$$
$$z_c = 0$$

$$z_0 = z_1 = (0, 0, 1)^T$$
$$O_0 = (0, 0, 0)^T$$
$$O_c = (a_1 C_1 + a_c C_{12}, a_1 S_1 + a_c S_{21}, 0)^T$$
$$O_1 = (a_1 C_1, a_1 S_1, 0)^T$$

$$z_0 \times (O_c - O_0) = (-a_1 S_1 - a_c S_{12}, a_1 C_1 + a_c C_{12}, 0)^T$$
$$z_1 \times (O_c - O_1) = (-a_c S_{12}, a_c C_{12}, 0)^T$$

$$\therefore \quad J = \begin{bmatrix} -a_1 S_1 - a_1 S_{12} & -a_c S_{12} & 0 \\ a_1 C_1 + a_c C_{12} & a_c C_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

5.2: Compute the Jacobian $J_{11}$ for the 3-link elbow manipulator of example 5.3.2 and show that it agrees with (5.3.14). Show that the determinant of this matrix agrees with (5.3.15)

Since all 3 joints are revolute:

$$J_{11} = \left[ z_0 \times (O_3 - O_0) \quad z_1 \times (O_3 - O_1) \quad z_2 \times (O_3 - O_2) \right]$$

$$O_0 = O_1 = (0,0,0)^T; \quad O_2 = \begin{bmatrix} a_2 c_1 c_2 \\ a_2 s_1 c_2 \\ a_2 s_2 \end{bmatrix}; \quad O_3 = \begin{bmatrix} a_2 c_1 c_2 + a_3 \\ a_2 s_1 c_2 + a_3 s_1 c_{23} \\ a_2 s_2 + a_3 s_{23} \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}; \quad z_2 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

Therefore

$$z_0 \times (O_3 - O_0) = \begin{bmatrix} -a_2 s_1 c_2 - a_3 s_1 c_{23} \\ a_2 c_1 c_2 + a_3 c_1 c_{23} \\ 0 \end{bmatrix}$$

$\cancel{z_1 \times}$

$$z_1 \times (O_3 - O_1) = \begin{bmatrix} -c_1 (a_2 s_2 + a_3 s_1 c_{23}) \\ -s_1 (a_2 s_2 + a_3 s_{23}) \\ a_2 c_2 + a_3 c_{23} \end{bmatrix}$$

$$z_2 \times (O_3 - O_2) = \begin{bmatrix} -a_3 c_1 s_{23} \\ -a_3 s_1 s_{23} \\ a_3 c_{23} \end{bmatrix}$$

$$\therefore J_{11} = \begin{bmatrix} -a_2 s_1 c_2 - a_3 s_1 c_{23} & -a_2 s_2 c_1 - a_3 s_{23} c_1 & -a_3 c_1 s_{23} \\ a_2 c_1 c_2 + a_3 c_1 c_{23} & -a_2 s_1 s_2 - a_3 s_1 s_{23} & -a_3 s_1 s_{23} \\ 0 & a_2 c_2 + a_3 c_{23} & a_3 c_{23} \end{bmatrix}$$

5.3

we have

$O_0 = 0$ ; $O_1 = (0, 0, d)^T$

$O_2 = (0, 0, d_1)^T$ ;

$O_3 = (-d_2 s_2 c_1, -d_2 s_2 s_1, d_1 + d_2 c_2)^T$

$z_0 = (0, 0, 1)^T$ ; $z_1 = (s_1, -c_1, 0)^T$ ; $z_2 = (-s_2 c_1, -s_2 s_1, c_2)^T$

$z_0 \times (O_3 - O_0) = (s_1 s_2 d_2, c_1 s_2 d_2, 0)^T$

$z_1 \times (O_3 - O_1) = (-c_1 c_2 d_2, s_1 c_2 d_2, s_2 d_2)^T$

$$\therefore J = \begin{bmatrix} s_1 s_2 d_2 & -c_1 c_2 d_2 & -s_2 c_1 \\ c_1 s_2 d_2 & s_1 c_2 d_2 & -s_2 s_1 \\ 0 & s_2 d_2 & c_2 \\ 0 & s_1 & \\ 0 & -c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

5.4 $d_1 d_4 - d_2 d_3 = (-a_1 s_1 - a_2 s_{12})(a_1 c_{12}) + (a_1 s_{12})(a_1 c_1 + a_2 c_{12})$

$= a_1^2 s_2$

5.5) $O_0 = (0,0,0)^T, \quad O_3 = (-d_3 s_1, d_3 c_1, 0)^T$

$z_0 = (0,0,1)^T; \quad z_1 = (0,0,1)^T, \quad z_2 = (-s_1, c_1, 0)^T$

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 & z_2 \\ z_0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -c_1 d_3 & 0 & -s_1 \\ -s_1 d_3 & 0 & c_1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\therefore \det \begin{bmatrix} -c_1 d_3 & 0 & -s_1 \\ -d_3 s_1 & 0 & c_1 \\ 0 & 1 & 0 \end{bmatrix} = c_1^2 d_3 + s_1^2 d_3 = d_3 \neq 0$$

5.6 All manipulators are prismatic

$$\therefore J = \begin{bmatrix} z_0 & z_1 & z_2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$