CHAPTER 5

5–1 Suppose a_c = distance from joint 2 to o_c, and a_e = length of link 1.

Then $\mathbf{o}_c = (x_c, y_c, z_c)^T$ where

$\mathbf{x}_c = a_1c_1 + a_cc_{12}$

$\mathbf{y}_c = a_1s_1 + a_cc_{12}$

$\mathbf{z}_c = 0.$

Also

$\mathbf{z}_0 = z_1 = (0,0,1)^T$

$\mathbf{o}_0 = (0,0,0)^T$

$\mathbf{o}_c = (a_1c_1 + a_cc_{12}, a_1s_1 + a_cs_{21}, 0)^T$

$\mathbf{o}_1 = (a_1c_1, a_1s_1, 0)^T$

$$\mathbf{z}_0 \times (\mathbf{o}_c - \mathbf{o}_0) = (-a_1s_1 - a_cs_{12}, a_1c_1 + a_cc_{12}, 0)^T$$

$$\mathbf{z}_1 \times (\mathbf{o}_c - \mathbf{o}_1) = (-a_cs_{12}, a_cc_{12}, 0)^T.$$

Therefore

$$J = \begin{bmatrix} -a_1s_1 - a_1s_{12} & -a_cs_{12} & 0 \\ a_1c_1 + a_cc_{12} & a_cc_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

5–2 Since all 3 joints are revolute,

$$J_{11} = [\mathbf{z}_0 \times (\mathbf{o}_3 - \mathbf{o}_0) \quad \mathbf{z}_1 \times (\mathbf{o}_3 - \mathbf{o}_1) \quad \mathbf{z}_2 \times (\mathbf{o}_3 - \mathbf{o}_2)]$$

$$\mathbf{o}_0 = \mathbf{o}_1 = (0,0,0)^T; \quad \mathbf{o}_2 = \begin{bmatrix} a_2c_1c_2 \\ a_2s_1c_2 \\ a_2s_2 \end{bmatrix}; \quad \mathbf{o}_3 = \begin{bmatrix} a_2c_1c_2 + a_3c_1c_{23} \\ a_2s_1c_2 + a_3s_1c_{23} \\ a_2s_2 + a_3s_{23} \end{bmatrix}$$

$$\mathbf{z}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad \mathbf{z}_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}; \quad \mathbf{z}_2 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}.$$

Therefore

$$\mathbf{z}_0 \times (\mathbf{o}_3 - \mathbf{o}_0) = \begin{bmatrix} -a_2s_1c_2 - a_3s_1c_{23} \\ a_2c_1c_2 + a_3c_1c_{23} \\ 0 \end{bmatrix}; \mathbf{z}_1 \times (\mathbf{o}_3 - \mathbf{o}_1) = \begin{bmatrix} -c_1(a_2s_2 + a_3s_1c_{23}) \\ -s_1(a_2s_2 + a_3s_{23}) \\ a_2c_2 + a_3c_{23} \end{bmatrix};$$

$$z_2 \times (o_3 - o_2) = \begin{bmatrix} -a_3c_1s_{23} \\ -a_3s_1s_{23} \\ a_3c_{23} \end{bmatrix}$$

and hence

$$J_{11} = \begin{bmatrix} -a_2s_1c_2 - a_3s_1c_{23} & -a_2s_2c_1 - a_3s_{23}c_1 & -a_3c_1s_{23} \\ a_2c_1c_2 + a_3c_1c_{23} & -a_2s_1s_2 - a_3s_1s_{23} & -a_3s_1s_{23} \\ 0 & a_2c_2 + a_3c_{23} & a_3c_{23} \end{bmatrix}$$

which agrees with (5.3.14). Next,

$$\begin{aligned} det\ J_{11} &= (-a_2s_1c_2 - a_3s_1c_{23})[(a_3c_{23})(-a_2s_1s_2 - a_3s_1s_{23}) + a_3s_1s_{23}(a_2c_2 + a_3c_{23})] - \\ &\quad (a_2c_1c_2 + a_3c_1c_{23})[(a_3c_{23})(-a_2s_2c_1 - a_3s_{23}c_1) + a_3c_1s_{23}(a_2c_2 + a_3c_{23})] \\ &= a_2^2a_3(s_2c_2c_{23} - s_{23}c_2^2) + a_2a_3^2(s_2c_{23}^2 - s_{23}c_2c_{23}) \\ &= -a_2^2a_3c_2s_3 - a_2a_3^2c_{23}s_3 \\ &= -a_2a_3s_3(a_2c_2 + a_3c_{23}). \end{aligned}$$

5–3

$$o_0 = o; \quad o_1 = (0,0,d_1)^T; \quad o_2 = (0,0,d_1)^T; \quad o_3 = (-d_2s_2c_1, -d_2s_2s_1, d_1 + d_2c_2)^T$$

$$z_0 = (0,0,1)^T; \quad z_1 = (s_1, -c_1, 0)^T; \quad z_2 = (-s_2c_1, -s_2s_1, c_2)^T$$

$$\begin{aligned} z_0 \times (o_3 - o_0) &= (s_1s_2d_2,\ c_1s_2d_2, 0)^T \\ z_1 \times (o_3 - o_1) &= (-c_1c_2d_2,\ s_1c_2d_2, s_2d_2)^T. \end{aligned}$$

Therefore

$$J = \begin{bmatrix} s_1s_2d_2 & -c_1c_2d_2 & -s_2c_1 \\ c_1s_2d_2 & s_1c_2d_2 & -s_2s_1 \\ 0 & s_2d_2 & c_2 \\ 0 & s_1 & 0 \\ 0 & -c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

5–4 From (5.3.18) the singularities are given by $\alpha_1\alpha_4 - \alpha_2\alpha_3 = 0$. From (5.3.19) we have

$$\begin{aligned} \alpha_1\alpha_4 - \alpha_2\alpha_3 &= (-a_1s_1 - a_2s_{12})(a_1c_{12}) + (a_1s_{12})(a_1c_1 + a_2c_{12}) \\ &= a_1^2s_2. \end{aligned}$$

which agrees with (5.3.20).

5–5 From Figure 3-7

$$o_0 = (0,0,0)^T; \quad o_3 = (-d_3s_1, d_3c_1, 0)^T$$

$$\mathbf{z}_0 = (0,0,1)^T; \quad \mathbf{z}_1 = (0,0,1)^T; \quad \mathbf{z}_2 = (-s_1, c_1, 0)^T$$

$$J = \begin{bmatrix} \mathbf{z}_0 \times (\mathbf{o}_3 - \mathbf{o}_0) & z_1 & z_2 \\ \mathbf{z}_0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -c_1 d_3 & 0 & -s_1 \\ -s_1 d_3 & 0 & c_1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Therefore

$$det \begin{bmatrix} -c_1 d_3 & 0 & -s_1 \\ -d_3 s_1 & 0 & c_1 \\ 0 & 1 & 0 \end{bmatrix} = c_1^2 d_3 + s_1^2 d_3 = d_3 \neq 0.$$

5–6 For cartesian manipulator, all joints are prismatic and hence

$$J = \begin{bmatrix} \mathbf{z}_0 & \mathbf{z}_1 & \mathbf{z}_2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{ which has rank 3.}$$

5–7

$$J = [J_1, \cdots, J_6]$$

where

$$J_1 = \begin{bmatrix} -d_y \\ d_x \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \quad J_2 = \begin{bmatrix} c_1 d_z \\ s_1 d_z \\ -s_1 d_y - c_1 d_x \\ -s_1 \\ c_1 \\ 0 \end{bmatrix}; \quad J_3 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$J_4 = \begin{bmatrix} s_1 s_2 (d_z - o_{3,z}) + c2(d_y - o_{3y}) \\ -c_1 s_1 (d_z - o_{3z}) + c_2 (d_x - o_{3x}) \\ -c_1 c_2 s_4 - s_1 c_4 \\ -s_1 c_2 s_4 + c_1 c_4 \\ s_2 s_4 \end{bmatrix}$$

$$J_5 = \begin{bmatrix} (-s_1 c_2 s_4 + c_1 c_4)(d_z - o_{3z}) - s_2 s_4 (d_y - o_{3y}) \\ (-c_1 c_2 s_4 + s_1 c_4)(d_z - o_{3z}) + s_2 s_4 (d_x - o_{3x}) \\ (-c_1 c_2 s_4 - s_1 c_4)(d_y - o_{3y}) + (s_1 c_2 s_4 - c_1 c_4)(d_x - o_{3x}) \\ -c_1 c_2 s_4 - s_1 c_4 \\ -s_1 c_2 s_4 + c_1 c_4 \\ s_2 s_4 \end{bmatrix}$$

$$J_6 = \begin{bmatrix} (s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5)(d_y - o_{3y}) + (s_2c_4s_5 - c_2c_5)(d_y - o_{3y}) \\ -(c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2c_5)(d_z - o_{3z}) + (s_2c_4s_5 - c_2c_5)(d_x - o_{3x}) \\ (c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2c_5)(d_y - o_{3y}) - (s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5)(d_y - o_{3x}) \\ c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2c_5 \\ s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 \\ -s_2c_4s_5 + c_2c_5 \end{bmatrix}$$

where

$$\mathbf{o}_6 = (d_x,\ d_y,\ d_z)^T$$

$$\mathbf{o}_3 = \begin{bmatrix} o_{3x} \\ o_{3y} \\ o_{3z} \end{bmatrix} = \begin{bmatrix} c_1s_2d_3 - s_1d_2 \\ s_1s_2d_3 + c_1d_2 \\ c_2d_3 \end{bmatrix}.$$