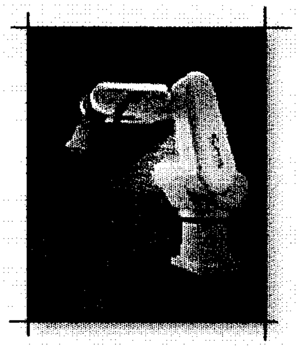
-20

Robotics, CpE 360



Assignment # 3

Submitted to: Prof. Sobh

Ghimire, Bishad
 # 0408455
Oct. 20, 1999

Q # 1. Verify the statement after equation (3.2 .6) that the rotation matrix R has the form (3.2 .4) provided assumptions DH1 and DH2 are satisfied

Solution :

According to equation #3 .2 .6, we know that R is given as :

```
<< LinearAlgebra`MatrixManipulation`

R = {{Cosθ, r12, r13}, {Sinθ, r22, r23}, {0, Sinα, Cosα}}; MatrixForm[R]
```

$$\begin{pmatrix} \text{Cos}\theta & r12 & r13 \\ \text{Sin}\theta & r22 & r23 \\ 0 & \text{Sin}\alpha & \text{Cos}\alpha \end{pmatrix}$$

But according to the ration on R,
we can expressed the R in terms of its columen i.e. when DH1 and DH2 are satisfied;

$$\begin{aligned} r^2 12 + r^2 22 &= 1 - \text{Sin}^2 \alpha \\ &= \text{Cos}^2 \alpha ; \\ \text{similarly : } r^2 13 + r^2 23 &= 1 - \text{Cos}^2 \alpha \\ &= \text{Sin}^2 \alpha . \end{aligned}$$

This yields, $r12 / \text{Cos}\alpha = -\text{Sin}\theta;$

$r22 / \text{Cos}\alpha = \text{Cos}\theta;$

$r13 / \text{Sin}\theta = \text{Sin}\theta;$

$r23 / \text{Sin}\alpha = -\text{Cos}\theta;$

This simply implies that equation (3.2 .4) i.e.

$$R = R_{z,\theta}\, R_{x,\alpha} = \begin{pmatrix} \text{Cos}\theta & -\text{Sin}\theta\text{Cos}\alpha & \text{Sin}\theta\text{Sin}\alpha \\ \text{Sin}\theta & \text{Cos}\theta\text{Cos}\alpha & -\text{Cos}\theta\text{Sin}\alpha \\ 0 & \text{Sin}\alpha & \text{Cos}\alpha \end{pmatrix}$$

where as our equation can be expressed after the substitutions :

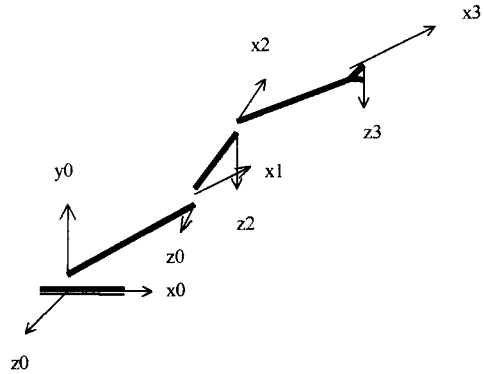
$$= \begin{pmatrix} \text{Cos}\theta & r12 & r13 \\ \text{Sin}\theta & r22 & r23 \\ 0 & \text{Sin}\alpha & \text{Cos}\alpha \end{pmatrix} = \begin{pmatrix} \text{Cos}\theta & -\text{Sin}\theta\text{Cos}\alpha & \text{Sin}\theta\text{Sin}\alpha \\ \text{Sin}\theta & \text{Cos}\theta\text{Cos}\alpha & -\text{Cos}\theta\text{Sin}\alpha \\ 0 & \text{Sin}\alpha & \text{Cos}\alpha \end{pmatrix}$$

Hence, the above mentioned condition has be verified.

Q # 2. Consider the three - link planar manipulator shown in fig .3 .12. Derive the forward kinematic equations using DH convention.

Solution :

From the figure 3.12, we can draw the skeleton as :



As we have a three - link planar manipulator, the link parameters can be given as :

link	ai	αi	di	θi
1	a1	0	0	θ1
2	a2	0	0	θ2
3	a3	0	0	θ3

It yields the matrix A as,

```
<< LinearAlgebra`MatrixManipulation`
```

The A - matices are determined to be the followings :

```
A1 = {{c1, -s1, 0, a1c1}, {s1, c2, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}; MatrixForm[A1]
```

$$\begin{pmatrix} c1 & -s1 & 0 & a1c1 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A2 = {{c2, -s2, 0, a2c2}, {s2, c2, 0, a2c2}, {0, 0, 1, 0}, {0, 0, 0, 1}};
MatrixForm[A2]
```

$$\begin{pmatrix} c2 & -s2 & 0 & a2c2 \\ s2 & c2 & 0 & a2c2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A3 = {{c3, -s3, 0, a3c3}, {s3, c3, 0, a3c3}, {0, 0, 1, 0}, {0, 0, 0, 1}};
MatrixForm[A3]
```

$$\begin{pmatrix} c3 & -s3 & 0 & a3c3 \\ s3 & c3 & 0 & a3s3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From the above A - matices, we can simply find the corresponding T - matices, i.e

$$T0^1 = A1$$

$$T0^2 = A1A2$$

$$T0^3 = A1A2A3 = \begin{pmatrix} c1 & -s1 & 0 & a1c1 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c2 & -s2 & 0 & a2c2 \\ s2 & c2 & 0 & a2c2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c3 & -s3 & 0 & a3c3 \\ s3 & c3 & 0 & a3s3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c123 & -s123 & 0 & a1c1 + a2c2 + a3c3 \\ s123 & c123 & 0 & a1s1 + a2s2 + a3s3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

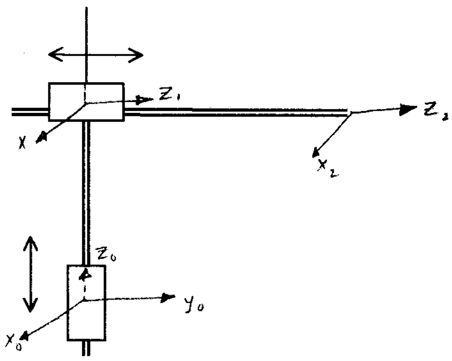
a2 s123

s12 s123

Thus, the forward kinematic equations using DH convention has been derived.

Q.# 3 Consider the two - link cartesian manipulator as shown below. Derives the forward kinematic equations using the DH convention.

<u>Solution :</u>



As we have a two - link planar cartesian manipulator, the link parameters can be given as :

link	ai	αi	di	θi
1	0	-90	d1	0
2	0	0	d2	0

It yields the matrix A as,

<< LinearAlgebra`MatrixManipulation`

The A - matices are determined to be the followings :

A1 = {{1, 0, 0, 0}, {0, 0, 1, 0}, {0, -1, 0, d1}, {0, 0, 0, 1}}; MatrixForm[A1]

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A2 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, d2}, {0, 0, 0, 1}}; MatrixForm[A2]

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From the above A - matices, we can simply find the corresponding T - matices, i.e

$$T0^1 = A1$$

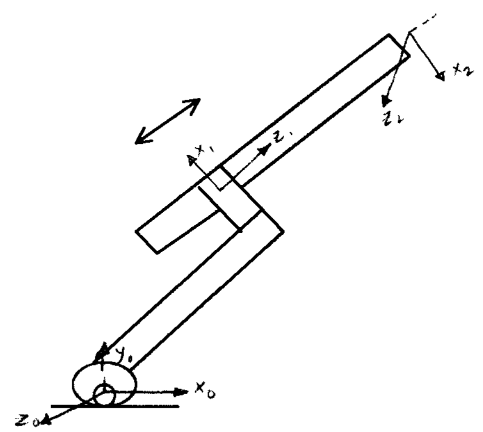
$$T0^2 = A1A2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d2 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, the forward kinematic equations using DH convention has been derived.

Q #4. Consider the two link manipulator from the fig below which has joint 1 revolute and joint 2 prismatic. Derive the forward kinematic equations using the DH - convention

Solution :



As we have a three - link planar manipulator, the link parameters can be given as :

link	ai	αi	di	θi
1	a1	90	0	θ1
2	0	90	d2	0

It yields the matrix A as,

```
<< LinearAlgebra`MatrixManipulation`
```

The A - matices are determined to be the followings :

```
A1 = {{c1, 0, s1, a1c1}, {s1, 0, -c1, a1s1}, {0, 1, 0, 0}, {0, 0, 0, 1}};
MatrixForm[A1]
```

$$\begin{pmatrix} c1 & 0 & s1 & a1c1 \\ s1 & 0 & -c1 & a1s1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A2 = {{1, 0, 0, 0}, {0, 0, 1, 0}, {0, -1, 0, d2}, {0, 0, 0, 1}}; MatrixForm[A2]
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From the above A - matices, we can simply find the corresponding T - matices, i.e

$T0^1 = A1$

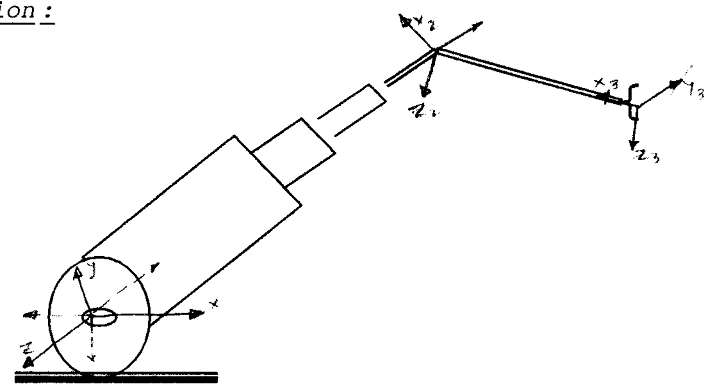
$$T0^2 = A1A2 = \begin{pmatrix} c1 & 0 & s1 & a1c1 \\ s1 & 0 & -c1 & a1s1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c1 & -s1 & 0 & a1c1 + d2s1 \\ s1 & c1 & 0 & a1s1 - d2c1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, the forward kinematic equations using DH convention has been derived.

Q #5. Consider the three - link planar manipulator from the fig below. Derive the forward kinematic equations using the DH - convention.

Solution :



As we have a three - link planar manipulator, the link parameters can be given as :

link	ai	αi	di	θi
1	0	90	0	θ1
2	0	-90	d2	0
3	a3	0	0	θ3

It yields the matrix A as,

```
<< LinearAlgebra`MatrixManipulation`
```

The A - matices are determined to be the followings :

```
A1 = {{c1, 0, s1, 0}, {s1, 0, -c1, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}}; MatrixForm[A1]
```

$$\begin{pmatrix} c1 & 0 & s1 & 0 \\ s1 & 0 & -c1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A2 = {{1, 0, 0, 0}, {0, 0, 1, 0}, {0, -1, 0, d2}, {0, 0, 0, 1}}; MatrixForm[A2]
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A2 = {{c3, -s3, 0, a3c3}, {s3, c3, 0, a3s3}, {0, 1, 1, d3}, {0, 0, 0, 1}};
MatrixForm[A2]
```

$$\begin{pmatrix} c3 & -s3 & 0 & a3c3 \\ s3 & c3 & 0 & a3s3 \\ 0 & 1 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From the above A - matices, we can simply find the corresponding T - matices, i.e

$T0^1$ = A1

$T0^2$ = A1A2

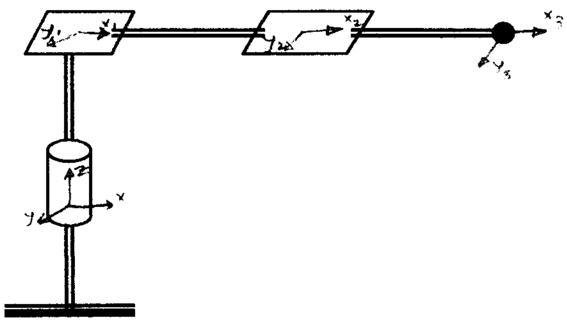
$$T0^3 = A1A2A3 = \begin{pmatrix} c1 & 0 & s1 & 0 \\ s1 & 0 & -c1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c3 & -s3 & 0 & a3c3 \\ s3 & c3 & 0 & a3s3 \\ 0 & 1 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c13 & -s13 & 0 & s1d2 + a3c13 \\ s13 & c13 & 0 & -c1d2 + a3s13 \\ 0 & 0 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, the forward kinematic equations using DH convention has been derived.

Q #6. Consider the three - link articulated robot as given below. Derive the forward kinematic equations using the DH - convention.

Solution :



As we have a three - link planar manipulator, the link parameters can be given as :

link	ai	αi	di	θi
1	0	90	0	θ1
2	a2	-0	0	θ2
3	a3	0	0	θ3

It yields the matrix A as,

```
<< LinearAlgebra`MatrixManipulation`
```

The A - matices are determined to be the followings :

```
A1 = {{c1, 0, s1, 0}, {s1, 0, -c1, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}}; MatrixForm[A1]
```

$$\begin{pmatrix} c1 & 0 & s1 & 0 \\ s1 & 0 & -c1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A2 = {{c2, -s2, 0, a2c2}, {s2, c2, 0, a2s2}, {0, -1, 1, 0}, {0, 0, 0, 1}};
MatrixForm[A2]
```

$$\begin{pmatrix} c2 & -s2 & 0 & a2c2 \\ s2 & c2 & 0 & a2s2 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A3 = {{c3, -s3, 0, a3c3}, {s3, c3, 0, a3s3}, {0, 1, 1, d3}, {0, 0, 0, 1}};
MatrixForm[A3]
```

$$\begin{pmatrix} c2 & -s2 & 0 & a2c2 \\ s2 & c2 & 0 & a2s2 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From the above A - matices, we can simply find the corresponding T - matices, i.e

$T0^1 = A1$

$T0^2 = A1A2$

$$T0^3 = A1A2A3 = \begin{pmatrix} c1 & 0 & s1 & 0 \\ s1 & 0 & -c1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c2 & -s2 & 0 & a2c2 \\ s2 & c2 & 0 & a2s2 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c2 & -s2 & 0 & a2c2 \\ s2 & c2 & 0 & a2s2 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

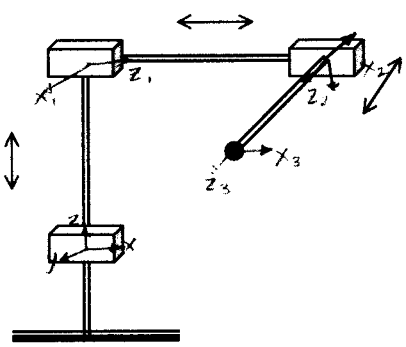
$$= \begin{pmatrix} c1c2c3 - c1s2s3 & -c1c2s3 - c1c2s2 & s1 & a2c1c2 + a3c1c2c3 - a3c1s2s3 \\ c2c3s1 - s1s2s3 & -c2s1s3 - c3s1s2 & -c1 & a2c2s1 + a3c2c3s1 - a3s1s2s3 \\ c2s3 + c3s2 & c2c3 - s2s3 & 0 & a2s2 + a3c2s3 + a3c3s2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Simply, } T0^3 = \begin{pmatrix} c1c23 & -c1s23 & s1 & a2c1c2 + a3s1c23 \\ s1c23 & -s1s23 & -c1 & a2c2s1 + a3s1c23 \\ s23 & c23 & 0 & a2s2 + a3s23 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, the forward kinematic equations using DH convention has been derived.

Q # 7. Consider the three - link cartesian manipulator as given below. Derive the forward kinematic equations using the DH - convention.

<u>Solution :</u>



As we have a three - link planar manipulator, the link parameters can be given as :

link	ai	αi	di	θi
1	0	-90	d1	0
2	0	90	d2	90
3	0	0	d3	-90

It yields the matrix A as,

<< LinearAlgebra`MatrixManipulation`

The A - matices are determined to be the followings :

```
A1 = {{1, 0, 0, 0}, {0, 0, 1, 0}, {0, -1, 0, d1}, {0, 0, 0, 1}}; MatrixForm[A1]
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A2 = {{0, 0, 1, 0}, {1, 0, 0, 0}, {0, 1, 0, d2}, {0, 0, 0, 1}}; MatrixForm[A2]
```

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A3 = {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 1, d3}, {0, 0, 0, 1}}; MatrixForm[A3]
```

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From the above A - matices, we can simply find the corresponding T - matices, i.e

$T0^1$ = A1

$T0^2$ = A1A2

$$\mathbf{T0^3 = A1A2A3} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

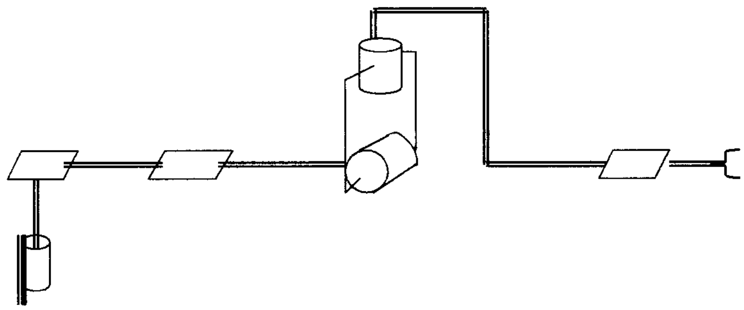
$$= \begin{pmatrix} 0 & 0 & 1 & d3 \\ -1 & 0 & 0 & d2 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

✓

Thus, the forward kinematic equations using DH convention has been derived.

Q #8 Attach a shperical wrist to the three - ink articulated manipulator from Q #6 as in Q #9. Derive the forward kinematic equation for the same.

Solution :



As we have a six - link planar manipulator, the link parameters can be given as :

link	ai	αi	di	θi
1	0	90	0	θ1
2	a2	0	0	θ2
3	a3	0	0	θ3
4	0	-90	0	θ4
5	0	0	0	θ5
6	0	0	d6	θ6

It yields the matrix A as,

```
<< LinearAlgebra`MatrixManipulation`
```

The A - matices are determined to be the followings :

```
A1 = {{c1, 0, s1, 0}, {s1, 0, -c1, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}}; MatrixForm[A1]
```

$$\begin{pmatrix} c1 & 0 & s1 & 0 \\ s1 & 0 & -c1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A2 = {{c2, -s2, 0, a2c2}, {s2, c2, 0, a2s2}, {0, 0, 1, 0}, {0, 0, 0, 1}};
MatrixForm[A2]
```

$$\begin{pmatrix} c2 & -s2 & 0 & a2c2 \\ s2 & c2 & 0 & a2s2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A3 = {{c3, -s3, 0, a3c3}, {s3, c3, 0, a3s3}, {0, 0, 1, 0}, {0, 0, 0, 1}};
MatrixForm[A3]
```

$$\begin{pmatrix} c3 & -s3 & 0 & a3c3 \\ s3 & c3 & 0 & a3s3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A4 = {{c4, 0, -s4, 0}, {s4 , 0, c4, 0}, {0, -1, 0, 0}, {0, 0, 0, 1}}; MatrixForm[A4]

$$\begin{pmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A5 = {{c5, 0, -s5, 0}, {s5 , 0, -c5, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}}; MatrixForm[A5]

$$\begin{pmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A5 = {{c6, -s6, 0, 0}, {s6 , c6, 0, 0}, {0, 0, 1, d6}, {0, 0, 0, 1}}; MatrixForm[A5]

$$\begin{pmatrix} c6 & -s6 & 0 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 1 & d6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From the above A - matices, we can simply find the corresponding T - matices, i.e

$T0^1$ = A1

$T0^2$ = A1A2

$T0^3$ = A1A2A3

$T0^4$ = A1A2A3A4

$T0^5$ = A1A2A3A4A5

$$T0^6 = A1A2A3A4A5A6 = \begin{pmatrix} c1 & 0 & s1 & 0 \\ s1 & 0 & -c1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c2 & -s2 & 0 & a2c2 \\ s2 & c2 & 0 & a2s2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} c3 & -s3 & 0 & a3c3 \\ s3 & c3 & 0 & a3s3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c6 & -s6 & 0 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 1 & d6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} c1[c5c6c234 - s6s234] - s1s5c6 & -c1[c5s6c234 + c6s234] + s1s5s6 & c1s5c234 + s1c5 \\ c1s5s6 + s1c5c6c234 - s1s6s234 & -c1s5s6 - s1c5s6c234 & -c1c5 + s1s5c234 \\ s6c234 + c5s6s234 & c6c234 - c5s6s234 & s5s234 \\ 0 & 0 & 0 \end{pmatrix}$$

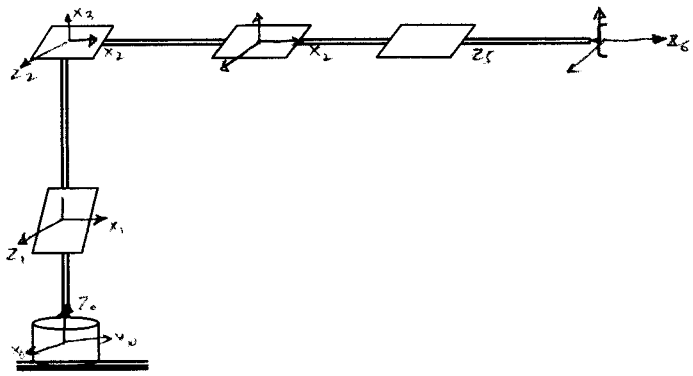
Where's rest?

-5

Thus, the forward kinematic equations using DH convention has been derived.

Q #13 Consider the GMF S - 400 robot shown in the fig below. Draw symbolic representation for this manipulator. Establish DH - coordinate frames and write the forward kinematic equations.

Solution :



As we have a six - link planar manipulator, the link parameters can be given as :

link	ai	αi	di	θi
1	0	90	d1	θ1
2	0	0	d2	θ2
3	0	90	0	θ3
4	0	-90	d4	θ4
5	0	90	0	θ5
6	0	0	d6	θ6

It yields the matrix A as,

```
<< LinearAlgebra`MatrixManipulation`
```

The A - matices are determined to be the followings :

```
A1 = {{c1, 0, s1, 0}, {s1, 0, -c1, 0}, {0, 1, 0, d1}, {0, 0, 0, 1}}; MatrixForm[A1]
```

$$\begin{pmatrix} c1 & 0 & s1 & 0 \\ s1 & 0 & -c1 & 0 \\ 0 & 1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A2 = {{c2, -s2, 0, 0}, {s2, c2, 0, 0}, {0, 1, 0, d2}, {0, 0, 0, 1}}; MatrixForm[A2]
```

$$\begin{pmatrix} c2 & -s2 & 0 & 0 \\ s2 & c2 & 0 & 0 \\ 0 & 0 & 1 & d2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A3 = {{c3, 0, s3, 0}, {s3, 0, -c3, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}}; MatrixForm[A3]
```

$$\begin{pmatrix} c3 & 0 & s3 & 0 \\ s3 & 0 & -c3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A4 = {{c4, 0, -s4, 0}, {s4, 0, c4, 0}, {0, -1, 0, d4}, {0, 0, 0, 1}}; MatrixForm[A4]
```

$$\begin{pmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & d4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A5 = {{c5, 0, s5, 0}, {s4, 0, -c5, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}}; MatrixForm[A5]

$$\begin{pmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A6 = {{c6, s6, 0, 0}, {s6, c6, 0, 0}, {0, 0, 1, d6}, {0, 0, 0, 1}}; MatrixForm[A6]

$$\begin{pmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From the above A - matices, we can simply find the corresponding T - matices, i.e

$T0^1$ = A1

$T0^2$ = A1A2

$T0^3$ = A1A2A3

$T0^4$ = A1A2A3A4

$T0^5$ = A1A2A3A4A5

$$T0^6 = A1A2A3A4A5A6 = \begin{pmatrix} c1 & 0 & s1 & 0 \\ s1 & 0 & -c1 & 0 \\ 0 & 1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c2 & -s2 & 0 & 0 \\ s2 & c2 & 0 & 0 \\ 0 & 0 & 1 & d2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} c3 & 0 & s3 & 0 \\ s3 & 0 & -c3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & d4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

It it simply expressed as :

```
c1c4c5c6c23 - c1s4s6c12 - c1c6s5s12 + s1 (c4s6 + s4c5c6)        -
 c1c4c5c6c23c23 - c1c6s4s23 + c1s5s6s23 + s1 (c4c6 - c5s4s6)
c4c5c6s1c23 - s1s4s6c23 - c6s1s5s23 - c1 (c4s6 + c5c6s4)         -
 c4c5s1s6s23 + c1 (c5s4s6 - c4c6)
c4c5c6s23 - s4s6s23 + c6s4s23                                     -
 c4c5s6s23 - s5s6c23 - c6s4s23
0                                                                 0


c1c4s5c23 + c1c5s23 + s1s4s5                           d2s1 +
 d4c1s23 + 6 (c1c4s5c23 + c1c5s23 + s1s4s5)
c4s1s5c23 + c5s1s23 - c1s4s5                           -
 d2c1 + d4s1s23 + d6 (-c4s1s5c23 + 5 s1)
c4s5s23 - c5s23                                        d1 -
 d4c23 + d6 (c4s5s23 - c5c23)
0                                                      1
```

Thus, the forward kinematic equations using DH convention has been derived.

Thus, the forward kinematic equations using DH convention has been derived.
