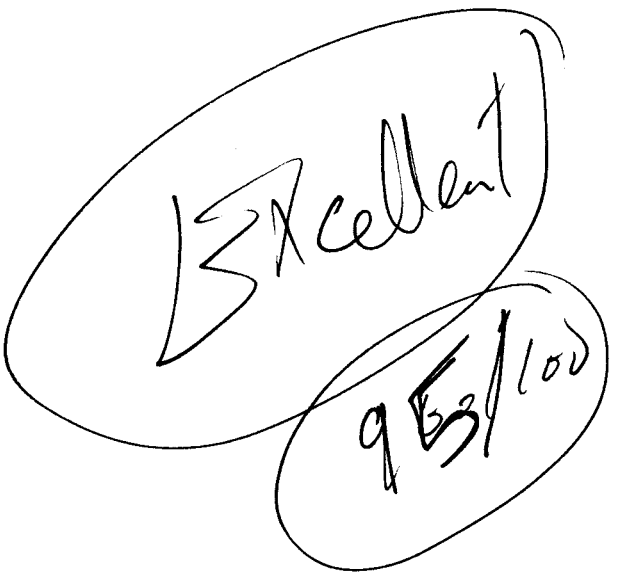
CpE360



Homework #3

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Robotics Homework #3

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Question 3.1

$$M = \begin{pmatrix} \text{Cos}[\theta] & a_{11} & a_{13} \\ \text{Sin}[\theta] & a_{22} & a_{23} \\ 0 & \text{Sin}[\alpha] & \text{Cos}[\alpha] \end{pmatrix}$$

{{Cos[θ], a_{11}, a_{13}}, {Sin[θ], a_{22}, a_{23}}, {0, Sin[α], Cos[α]}}

Hence

```
M[[1, 1]]^2 + M[[3, 1]]^2
```

$\text{Cos}[\theta]^2$

```
M[[3, 1]]^2 + M[[3, 2]]^2
```

$\text{Sin}[\alpha]^2$

```
M[[2, 1]] / Cos[α]
```

Sec[α] Sin[θ]

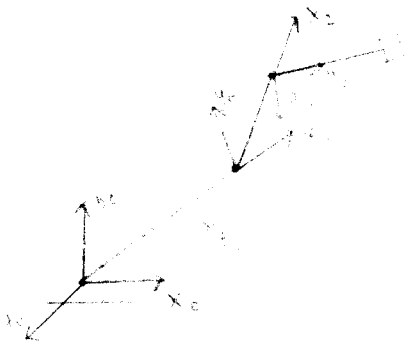
```
M[[3, 1]] / Cos[α]
```

0

```
M[[3, 2]] / Cos[α]
```

```
Tan[α]
```

Question 3.2



Below is the D - H Parameter table :

link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

These are the transformation matrices

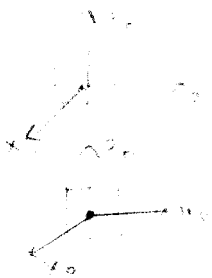
$$M1 = \begin{pmatrix} \text{Cos}[\theta_1] & -\text{Sin}[\theta_1] & 0 & a_1 * \text{Cos}[\theta_1] \\ \text{Sin}[\theta_1] & \text{Cos}[\theta_1] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$M2 = \begin{pmatrix} \text{Cos}[\theta_2] & -\text{Cos}[\theta_2] & 0 & a_2 * \text{Cos}[\theta_2] \\ \text{Sin}[\theta_2] & \text{Cos}[\theta_2] & 0 & a_2 * \text{Cos}[\theta_2] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad M3 = \begin{pmatrix} \text{Cos}[\theta_3] & -\text{Sin}[\theta_3] & 0 & a_3 * \text{Cos}[\theta_3] \\ \text{Sin}[\theta_3] & \text{Cos}[\theta_3] & 0 & a_3 * \text{Sin}[\theta_3] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$T_{0,3}$ = M1.M2.M3

```
{{Cos[θ3] (Cos[θ1] Cos[θ2] - Sin[θ1] Sin[θ2]) + (-Cos[θ1] Cos[θ2] - Cos[θ2] Sin[θ1]) Sin[θ3],
  Cos[θ3] (-Cos[θ1] Cos[θ2] - Cos[θ2] Sin[θ1]) - (Cos[θ1] Cos[θ2] - Sin[θ1] Sin[θ2]) Sin[θ3],
  0, Cos[θ1] a1 + Cos[θ1] Cos[θ2] a2 - Cos[θ2] Sin[θ1] a2 +
   Cos[θ3] (Cos[θ1] Cos[θ2] - Sin[θ1] Sin[θ2]) a3 +
   (-Cos[θ1] Cos[θ2] - Cos[θ2] Sin[θ1]) Sin[θ3] a3},
 {Cos[θ3] (Cos[θ2] Sin[θ1] + Cos[θ1] Sin[θ2]) + (Cos[θ1] Cos[θ2] - Cos[θ2] Sin[θ1]) Sin[θ3],
  Cos[θ3] (Cos[θ1] Cos[θ2] - Cos[θ2] Sin[θ1]) - (Cos[θ2] Sin[θ1] + Cos[θ1] Sin[θ2]) Sin[θ3],
  0, Cos[θ1] Cos[θ2] a2 + Cos[θ2] Sin[θ1] a2 + Cos[θ3] (Cos[θ2] Sin[θ1] + Cos[θ1] Sin[θ2]) a3 +
   (Cos[θ1] Cos[θ2] - Cos[θ2] Sin[θ1]) Sin[θ3] a3}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

Question 3.3



Below is the D - H Parameter table :

link	a_i	α_i	d_i	θ_i
1	0	-90°	d_1	0
2	0	0	d_2	0

These are the transformation matrices

$$M1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$M2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

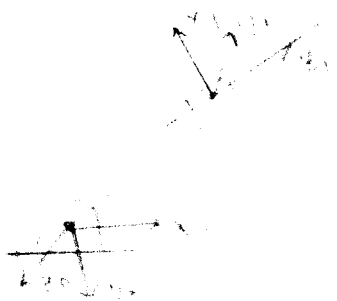
$T_{0,2}$ = M1.M2

```
{{1, 0, 0, 0}, {0, 0, 1, d2}, {0, -1, 0, d1}, {0, 0, 0, 1}}
```

```
MatrixForm[%]
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Question 3.4



Below is the D - H Parameter table :

link	a_i	α_i	d_i	θ_i
1	a_1	90 °	0	θ_1
2	0	90 °	d_2	0

These are the transformation matrices

```
M1 = {{Cos[θ1], 0, Sin[θ1], a1 * Cos[θ1]},
      {Sin[θ1], 0, Cos[θ1], a1 * Sin[θ1]},
      {0, 1, 0, 0}, {0, 0, 0, 1}};
```

$$M1 = \begin{pmatrix} \cos[\theta_1] & 0 & \sin[\theta_1] & a_1 * \cos[\theta_1] \\ \sin[\theta_1] & 0 & \cos[\theta_1] & a_1 * \sin[\theta_1] \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

```
M2 = {{1, 0, 0, 0}, {0, 0, 1, 0}, {0, -1, 0, d2}, {0, 0, 0, 1}};
```

$$M2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

```
T0,2 = M1.M2
```

$$T_{0,2} = M1.M2$$

```
{{Cos[θ1], -Sin[θ1], 0, Cos[θ1] a1 + Sin[θ1] d2},
 {Sin[θ1], -Cos[θ1], 0, Sin[θ1] a1 + Cos[θ1] d2}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

MatrixForm[%]

$$\begin{pmatrix} \text{Cos}[\theta_1] & -\text{Sin}[\theta_1] & 0 & \text{Cos}[\theta_1]\, a_1 + \text{Sin}[\theta_1]\, d_2 \\ \text{Sin}[\theta_1] & -\text{Cos}[\theta_1] & 0 & \text{Sin}[\theta_1]\, a_1 + \text{Cos}[\theta_1]\, d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Question 3.5

Below is the D - H Parameter table :

link	a_i	α_i	d_i	θ_i
1	0	90 °	0	θ_1
2	0	-90 °	d_2	0
3	a_3	0	0	θ_3

These are the transformation matrices

$$\mathbf{M1} = \begin{pmatrix} \text{Cos}[\theta_1] & 0 & \text{Sin}[\theta_1] & 0 \\ \text{Sin}[\theta_1] & 0 & -\text{Cos}[\theta_1] & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{M2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{M3} = \begin{pmatrix} \text{Cos}[\theta_3] & -\text{Sin}[\theta_3] & 0 & a_3 * \text{Cos}[\theta_3] \\ \text{Sin}[\theta_3] & \text{Cos}[\theta_3] & 0 & a_3 * \text{Sin}[\theta_3] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

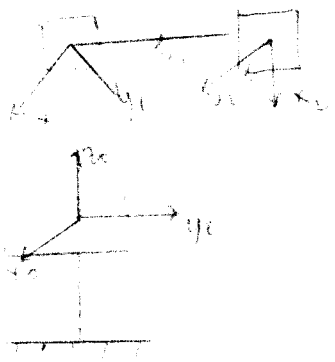
$T_{0,3}$ = M1.M2.M3

```
{{Cos[θ1] Cos[θ3] - Sin[θ1] Sin[θ3], -Cos[θ3] Sin[θ1] - Cos[θ1] Sin[θ3],
  0, Cos[θ1] Cos[θ3] a3 - Sin[θ1] Sin[θ3] a3 + Sin[θ1] d2},
 {Cos[θ3] Sin[θ1] + Cos[θ1] Sin[θ3], Cos[θ1] Cos[θ3] - Sin[θ1] Sin[θ3], 0,
  Cos[θ3] Sin[θ1] a3 + Cos[θ1] Sin[θ3] a3 - Cos[θ1] d2}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

MatrixForm[%]

$$\begin{pmatrix} \text{Cos}[\theta_1]\,\text{Cos}[\theta_3] - \text{Sin}[\theta_1]\,\text{Sin}[\theta_3] & -\text{Cos}[\theta_3]\,\text{Sin}[\theta_1] - \text{Cos}[\theta_1]\,\text{Sin}[\theta_3] & 0 & \text{Cos}[\theta_1]\,\text{Cos}[\theta_3]\,a_3 - \text{Sin}[\theta_1]\,\text{Sin}[\theta \\ \text{Cos}[\theta_3]\,\text{Sin}[\theta_1] + \text{Cos}[\theta_1]\,\text{Sin}[\theta_3] & \text{Cos}[\theta_1]\,\text{Cos}[\theta_3] - \text{Sin}[\theta_1]\,\text{Sin}[\theta_3] & 0 & \text{Cos}[\theta_3]\,\text{Sin}[\theta_1]\,a_3 + \text{Cos}[\theta_1]\,\text{Sin}[\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Question 3.7



Below is the D - H Parameter table :

link	a_i	α_i	d_i	θ_i
1	0	-90 °	d_1	0
2	0	90 °	d_2	-90 °
3	0	0	d_3	90 °

These are the transformation matrices

$$\mathbf{M1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{M2} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{M3} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$T_{0,3}$ = M1.M2.M3

$\{\{0, 0, 1, d_3\}, \{1, 0, 0, d_2\}, \{0, 1, 0, d_1\}, \{0, 0, 0, 1\}\}$

```
MatrixForm[%]
```

$$\begin{pmatrix} 0 & 0 & 1 & d_3 \\ 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Question 3.8

Below is the D - H Parameter table :

link	a_i	α_i	d_i	θ_i
1	0	90 °	0	θ_1
2	a_2	0 °	0	θ_2
3	a_3	0	0	θ_3
4	0	90 °	0	θ_4
5	0	0	0	θ_5
6	0	0	d_6	θ_6

These are the transformation matrices

$$\mathbf{M1} = \begin{pmatrix} \mathrm{Cos}[\theta_1] & 0 & \mathrm{Sin}[\theta_1] & 0 \\ \mathrm{Sin}[\theta_1] & 0 & -\mathrm{Cos}[\theta_1] & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{M2} = \begin{pmatrix} \mathrm{Cos}[\theta_2] & -\mathrm{Cos}[\theta_2] & 0 & a_2 * \mathrm{Cos}[\theta_2] \\ \mathrm{Sin}[\theta_2] & \mathrm{Cos}[\theta_2] & 0 & a_2 * \mathrm{Cos}[\theta_2] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{M3} = \begin{pmatrix} \mathrm{Cos}[\theta_3] & -\mathrm{Sin}[\theta_3] & 0 & a_3 * \mathrm{Cos}[\theta_3] \\ \mathrm{Sin}[\theta_3] & \mathrm{Cos}[\theta_3] & 0 & a_3 * \mathrm{Sin}[\theta_3] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{M4} = \begin{pmatrix} \mathrm{Cos}[\theta_4] & 0 & \mathrm{Sin}[\theta_4] & 0 \\ \mathrm{Sin}[\theta_4] & 0 & -\mathrm{Cos}[\theta_4] & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{M5} = \begin{pmatrix} \mathrm{Cos}[\theta_5] & 0 & \mathrm{Sin}[\theta_5] & 0 \\ \mathrm{Sin}[\theta_5] & 0 & -\mathrm{Cos}[\theta_5] & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$\mathbf{M6} = \begin{pmatrix} \mathrm{Cos}[\theta_6] & 0 & \mathrm{Sin}[\theta_6] & 0 \\ \mathrm{Sin}[\theta_6] & 0 & -\mathrm{Cos}[\theta_6] & 0 \\ 0 & 1 & 0 & d_6 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$T_{0,6}$ = M1.M2.M3.M4.M5.M6

```
{{Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) +
          (-Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ4]) + Sin[θ1] Sin[θ5]) +
    (-Cos[θ4] (-Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) +
       (Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ4]) Sin[θ6],
  -Cos[θ5] Sin[θ1] + (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) +
       (-Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ4]) Sin[θ5],
  -Cos[θ6] (-Cos[θ4] (-Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) +
       (Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ4]) +
    (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) +
          (-Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ4]) + Sin[θ1] Sin[θ5])
     Sin[θ6], Cos[θ1] Cos[θ2] a2 + Cos[θ1] Cos[θ2] Cos[θ3] a3 - Cos[θ1] Cos[θ2] Sin[θ3] a3 +
    (-Cos[θ5] Sin[θ1] + (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) +
          (-Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ4]) Sin[θ5]) d6},
 {Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) +
          (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ4]) - Cos[θ1] Sin[θ5]) +
    (-Cos[θ4] (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) +
       (Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ4]) Sin[θ6],
  Cos[θ1] Cos[θ5] + (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) +
       (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ4]) Sin[θ5],
  -Cos[θ6] (-Cos[θ4] (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) +
       (Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ4]) +
    (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) +
          (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ4]) - Cos[θ1] Sin[θ5])
     Sin[θ6], Cos[θ2] Sin[θ1] a2 + Cos[θ2] Cos[θ3] Sin[θ1] a3 - Cos[θ2] Sin[θ1] Sin[θ3] a3 +
    (Cos[θ1] Cos[θ5] + (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) +
          (-Cos[θ2] Cos[θ3] Sin[θ1] - Cos[θ2] Sin[θ1] Sin[θ3]) Sin[θ4]) Sin[θ5]) d6},
 {Cos[θ5] Cos[θ6] (Cos[θ4] (Cos[θ3] Sin[θ2] + Cos[θ2] Sin[θ3]) +
       (Cos[θ2] Cos[θ3] - Sin[θ2] Sin[θ3]) Sin[θ4]) +
    (-Cos[θ4] (Cos[θ2] Cos[θ3] - Sin[θ2] Sin[θ3]) +
       (Cos[θ3] Sin[θ2] + Cos[θ2] Sin[θ3]) Sin[θ4]) Sin[θ6],
  (Cos[θ4] (Cos[θ3] Sin[θ2] + Cos[θ2] Sin[θ3]) +
     (Cos[θ2] Cos[θ3] - Sin[θ2] Sin[θ3]) Sin[θ4]) Sin[θ5],
  -Cos[θ6] (-Cos[θ4] (Cos[θ2] Cos[θ3] - Sin[θ2] Sin[θ3]) +
       (Cos[θ3] Sin[θ2] + Cos[θ2] Sin[θ3]) Sin[θ4]) +
    Cos[θ5] (Cos[θ4] (Cos[θ3] Sin[θ2] + Cos[θ2] Sin[θ3]) +
       (Cos[θ2] Cos[θ3] - Sin[θ2] Sin[θ3]) Sin[θ4]) Sin[θ6],
  Cos[θ2] a2 + Cos[θ3] Sin[θ2] a3 + Cos[θ2] Sin[θ3] a3 +
    (Cos[θ4] (Cos[θ3] Sin[θ2] + Cos[θ2] Sin[θ3]) +
       (Cos[θ2] Cos[θ3] - Sin[θ2] Sin[θ3]) Sin[θ4]) Sin[θ5] d6}, {0, 0, 0, 1}}
```

Question 3.9

$T_{0,6} = T_{0,3} T_{3,6}$

Hence we get the transformation matrix

$T_{0,6}$