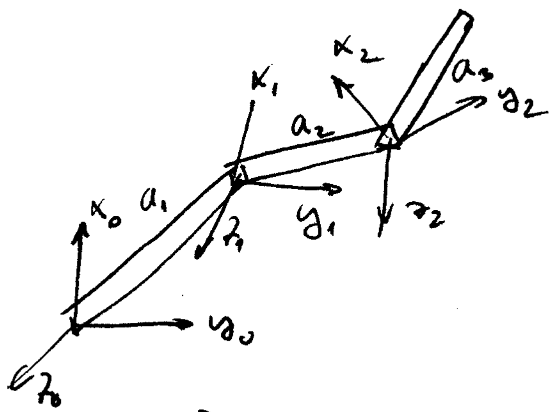
Homework 3

Andrew Rosca

Please check Soln = Key

-30

3-2



	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

$$A_i = \begin{bmatrix} C_i & -S_i & 0 & a_iC_i \\ S_i & C_i & 0 & a_iS_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow A_1A_2 = \begin{bmatrix} C_{12} & -S_{12} & 0 & a_1C_1+a_2C_{12} \\ S_{12} & C_{12} & 0 & a_1S_1+a_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

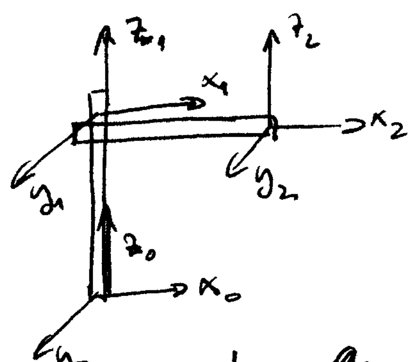
$$\Rightarrow A_1A_2A_3 = \begin{bmatrix} C_{12} & -S_{12} & 0 & a_1C_1+a_2C_{12} \\ S_{12} & C_{12} & 0 & a_1S_1+a_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} C_{123} & -S_{123} & 0 & a_1C_1+a_2C_{12}+a_3C_{123} \\ S_{123} & C_{123} & 0 & a_1S_1+a_2S_{12}+a_3S_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_0^3$$

$$x = a_1C_1 + a_2C_{12} + a_3C_{123}$$
$$y = a_1S_1 + a_2S_{12} + a_3S_{123}$$
$$z = 0$$

$$R = \begin{bmatrix} C_{123} & -S_{123} & 0 \\ S_{123} & C_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3-3



$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

	a_i	α_i	d_i	θ_i
1	0	0	d_1	0
2	a_2	0	0	0

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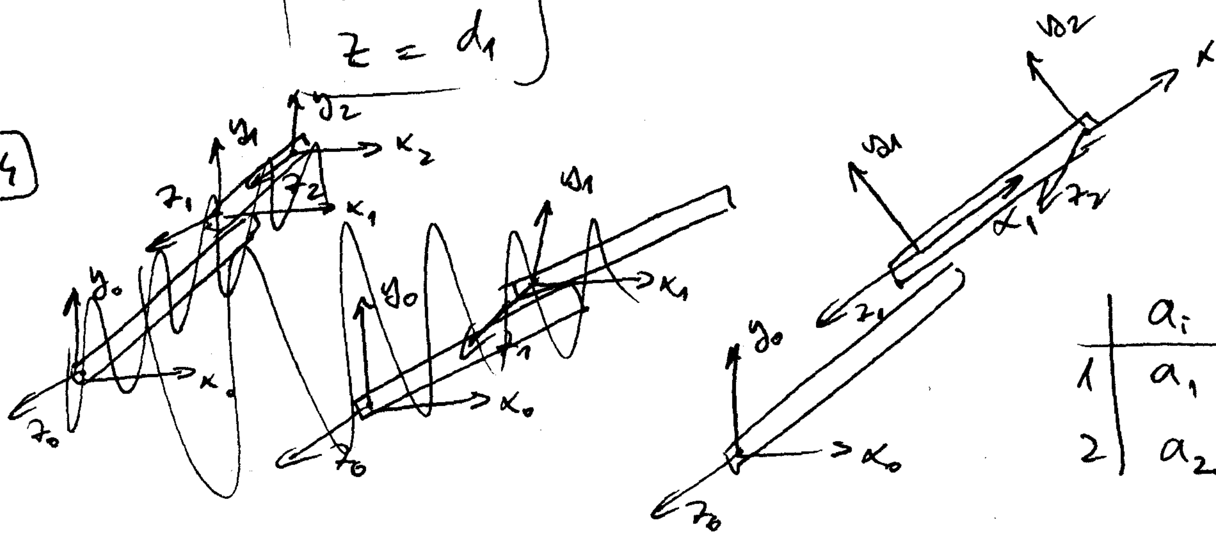
$$A_2 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T$$

$$\boxed{\begin{aligned} x &= a_2 \\ y &= 0 \\ z &= d_1 \end{aligned}} \qquad R = 0$$

$\boxed{3-4}$



	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	0

$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & a_1 C_1 \\ S_1 & C_1 & 0 & a_1 S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

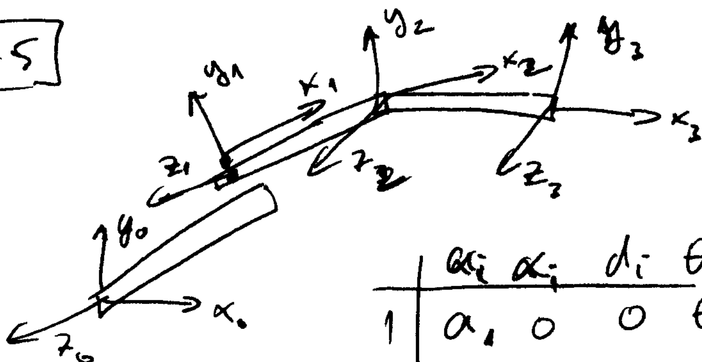
$$A_2 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & -s_1 & 0 & c_1(a_1+a_2) \\ s_1 & c_1 & 0 & s_1(a_1+a_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = c_1\,(a_1 + a_2)$$
$$y = s_1\,(a_1 + a_2)$$
$$z = 0$$

$$R = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{3-5}$



	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	0
3	a_3	0	0	θ_3

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 \cancel{A_3} = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & -s_1 & 0 & c_1(a_1+a_2) \\ s_1 & c_1 & 0 & s_1(a_1+a_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} \cancel{c_{13} - s_{13}} & \cancel{c_3 s_1 - s_3 c_1} & \cancel{0} & \cancel{(c_{13} - s_{13})(a_1 + a_2) + a_3 c_3} \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

$$A_1A_2A_3 = \begin{bmatrix} c_1 & -s_1 & 0 & c_1(a_1+a_2) \\ s_1 & c_1 & 0 & s_1(a_1+a_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & c_3 a_3 \\ s_3 & c_3 & 0 & s_3 a_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} c_{13} & -(c_1 s_3 + c_3 s_1) & 0 & c_{13} a_3 + c_1(a_1+a_2) \\ c_1 s_3 + c_3 s_1 & c_{13} & 0 & s_{13} a_3 + s_1(a_1+a_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

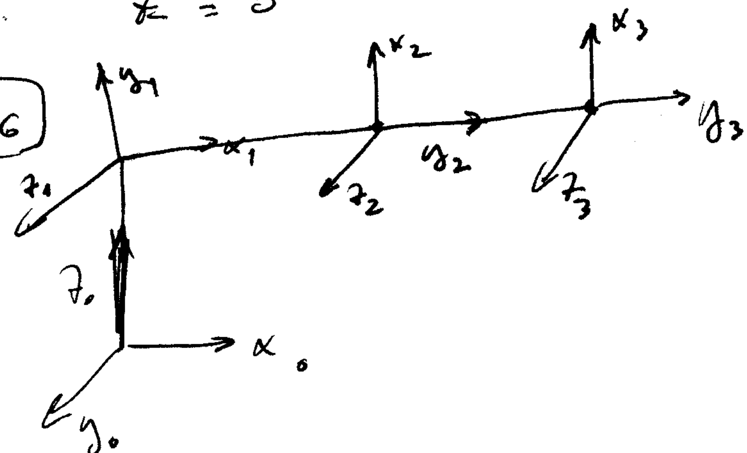
$$x = c_{13} a_3 + c_1(a_1+a_2)$$

$$y = s_{13} a_3 + s_1(a_1+a_2)$$

$$z = 0$$

$$R = \begin{bmatrix} c_{13} & -(c_1 s_3 + c_3 s_1) & 0 \\ c_1 s_3 + c_3 s_1 & c_{13} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{3-6}$



	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

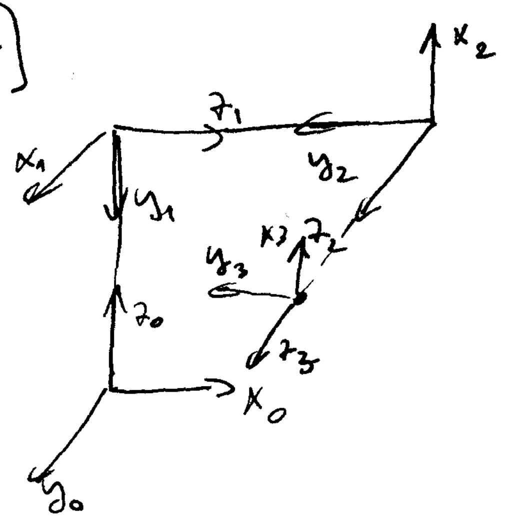
$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & c_{12} a_2 \\ s_{12} & c_{12} & 0 & s_{12} a_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} c_{123} & -s_{123} & 0 & c_{12} a_2 + c_{123} a_3 \\ s_{123} & c_{123} & 0 & s_{12} a_2 + s_{123} a_3 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{3-7}$



	a_i	α_i	d_i	θ_i
1	0	0	d_1	0
2	0	90	d_2	0
3	0	90	d_3	0

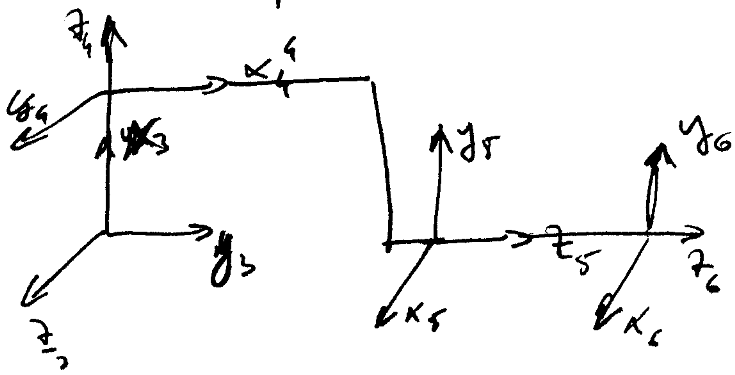
$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & d_1 + d_2 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boxed{3-8}$ first part is identical to $\boxed{3-6}$

second part:



	a_i	α_i	d_i	θ_i
4	a_4	α_4	0	0
5	a_5	0	$-d_5$	θ_5
6	0	0	d_6	θ_6

$$A_4 = \begin{bmatrix} 1 & 0 & 0 & a_4 \\ 0 & c_4 & -s_4 & 0 \\ 0 & s_4 & c_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_5 = \begin{bmatrix} c_5 & -s_5 & 0 & c_5 a_5 \\ s_5 & c_5 & 0 & s_5 a_5 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_4 A_5 = \begin{bmatrix} c_5 & -s_5 & 0 & c_5 a_5 + a_4 \\ s_5 & c_4 c_5 & -s_4 & s_5 a_5 + a_4 \\ 0 & s_4 & c_4 c_5 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 A_5 A_6 = \begin{bmatrix} c_{56} & -s_{56} & 0 & c_5 a_5 + a_4 \\ s_{56} & c_4 c_{56} & -s_4 & s_5 a_5 + a_4 \\ 0 & s_4 & c_4 c_{56} & d_5 + d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_1 A_2 A_3 A_4 A_5 A_6 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{56} & -s_{56} & 0 & c_5 a_5 + a_4 \\ s_{56} & c_4 c_{56} & -s_4 & s_5 a_5 + a_4 \\ 0 & s_4 & c_4 c_{56} & d_5 + d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$