CHAPTER 3

3–1 From Equation (3.2.6) we know that R has the form

$$R = \begin{bmatrix} c_\theta & r_{12} & r_{13} \\ s_\theta & r_{22} & r_{23} \\ 0 & s_\alpha & c_\alpha \end{bmatrix}$$

Since R is a rotation matrix the column vectors satisfy

$$r_{12}^2 + r_{22}^2 = 1 - s_\alpha^2 = c_\alpha^2$$
$$r_{13}^2 + r_{23}^2 = 1 - c_\alpha^2 = s_\alpha^2$$

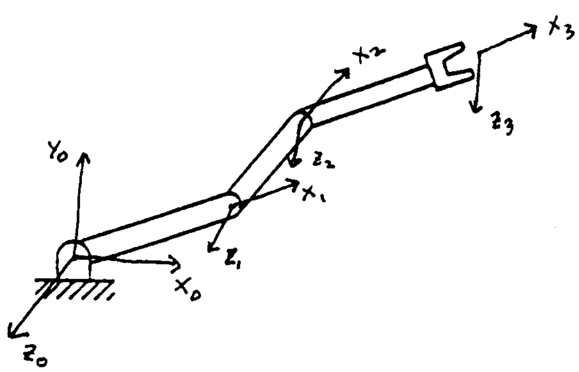
Therefore there is a unique angle θ such that

$$r_{12}/c_\alpha = -s_\theta \; ; \quad r_{22}/c_\alpha = c_\theta$$
$$r_{13}/s_\alpha = s_\theta \quad ; \quad r_{23}/s_\alpha = -c_\theta$$

and the result follows.

In each of the following problems 3-2 to 3-7, the figure shows the DH-coordinate frames. Then a table of DH-parameters is given followed by the corresponding A matrices and the T matrix giving the transformation between the base frame and the end-effector frame.

3–2

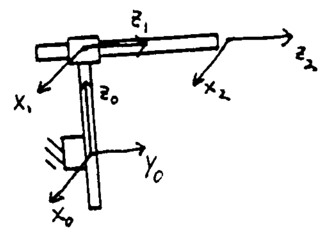


link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \; A_2 = \begin{bmatrix} c_2 & -c_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \; A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ 3_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1\ A_2\ A_3 = \begin{bmatrix} c_{123} & -s_{123} & 0 & a_1c_1 + a_2c_{12} + a_3c_{123} \\ s_{123} & c_{123} & 0 & a_1s_1 + a_2s_{12} + a_3s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3–3

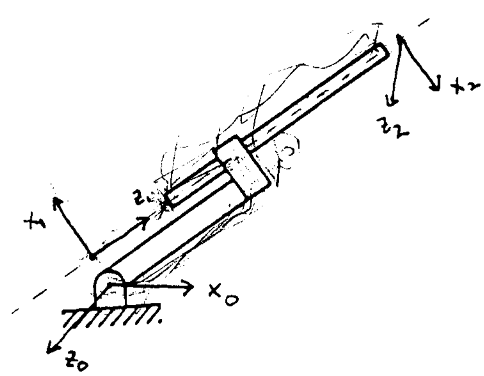


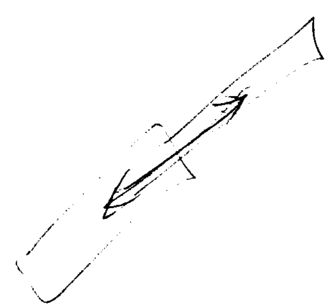
link	a_i	α_i	d_i	θ_i
1	0	$-90°$	d_1	0
2	0	0	d_2	0

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ; \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^2 = A_1A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3–4



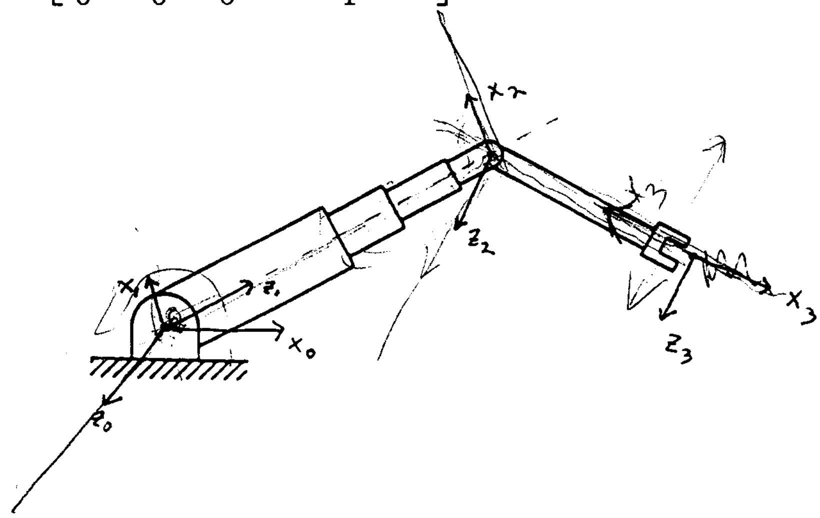


link	a_i	α_i	d_i	θ_i
1	a_1	90	0	θ_1
2	0	90	d_2	0

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & a_1c_1 \\ s_1 & 0 & -c_1 & a_1s_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^2 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 + d_2s_1 \\ s_1 & c_1 & 0 & a_1s_1 - d_2c_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3–5

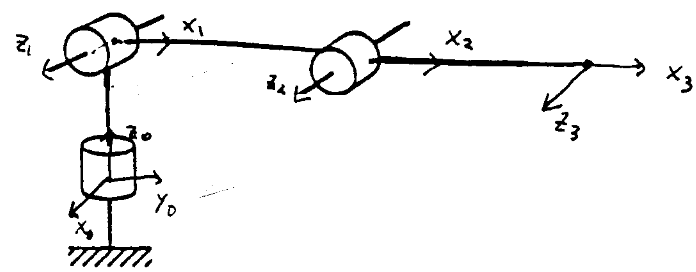


link	a_i	α_i	d_i	θ_i
1	0	90°	0	θ_1
2	0	−90°	d_2	0
3	a_3	0	$\cancel{d_3}$ 0	θ_3

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1A_2A_3 = \begin{bmatrix} c_{13} & -s_{13} & 0 & s_1d_2 + a_3c_{13} \\ s_{13} & c_{13} & 0 & -c_1d_2 + a_3s_{13} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3–6



link	a_i	α_i	d_i	θ_i
1	0	90	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where:

$r_{11} = c_1c_2c_3 - c_1s_2s_3 = c_1c_{23}$

$r_{12} = -c_1c_2s_3 - c_1c_3s_2 = -c_1s_{23}$

$r_{13} = s_1$

$d_x = a_2c_1c_2 + a_3c_1c_2c_3 - a_3c_1s_2s_3 = a_2c_1c_2 + a_3c_1c_{23}$

$r_{21} = c_2c_3s_1 - s_1s_2s_3 = s_1c_{23}$

$r_{22} = -c_2s_1s_3 - c_3s_1s_2 = -s_1s_{23}$

$r_{23} = -c_1$

$d_y = a_2c_2s_1 + a_3c_2c_3s_1 - a_3s_1s_2s_3 = a_2c_2s_1 + a_3s_1c_{23}$

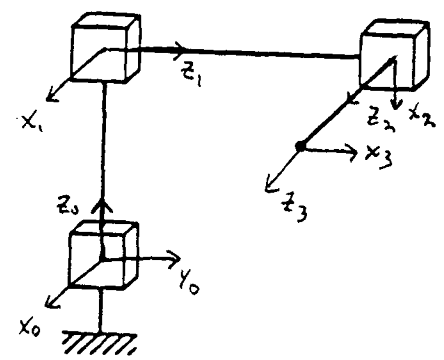
$r_{31} = c_2s_3 + c_3s_2 = s_{23}$

$r_{32} = c_2c_3 - s_2s_3 = c_{23}$

$r_{33} = 0$

$d_z = a_2s_2 + a_3c_2s_3 + a_3c_3s_2 = a_2s_2 + a_3s_{23}$

3–7



link	a_i	α_i	d_i	θ_i
1	0	$-90°$	d_1	0
2	0	$90°$	d_2	$90°$
3	0	0	d_3	$-90°$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1\ A_2\ A_3 \ = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ -1 & 0 & 0 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3–8

link	a_i	α_i	d_i	θ_i
1	0	$90°$	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	$-90°$	0	θ_4
5	0	0	0	θ_5
6	0	0	d_6	θ_6

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
r_{11} &= c_1[c_5c_6c_{234} - s_6s_{234}] - s_1s_5c_6 \\
r_{12} &= -c_1[c_5s_6c_{234} + c_6s_{234}] + s_1s_5s_6 \\
r_{13} &= c_1s_5c_{234} + s_1c_5 \\
d_x &= a_2c_1c_2 + a_3c_1c_{23} + d_6[c_1s_5c_{234} + s_1c_5] \\
r_{21} &= c_1s_5s_6 + s_1c_5c_6c_{234} - s_1s_6s_{234} \\
r_{22} &= -c_1s_5s_6 - s_1c_5s_6c_{234} \\
r_{23} &= -c_1c_5 + s_1s_5c_{234} \\
d_y &= a_2s_1c_2 + a_3s_1c_{23} - d_6[c_1c_5 + s_1s_5c_{234}] \\
r_{31} &= s_6c_{234} + c_5s_6s_{234} \\
r_{32} &= c_6c_{234} - c_5s_6s_{234} \\
r_{33} &= s_5s_{234} \\
d_z &= a_2s_2 + a_3s_{23} + d_6s_5s_{234}
\end{aligned}$$

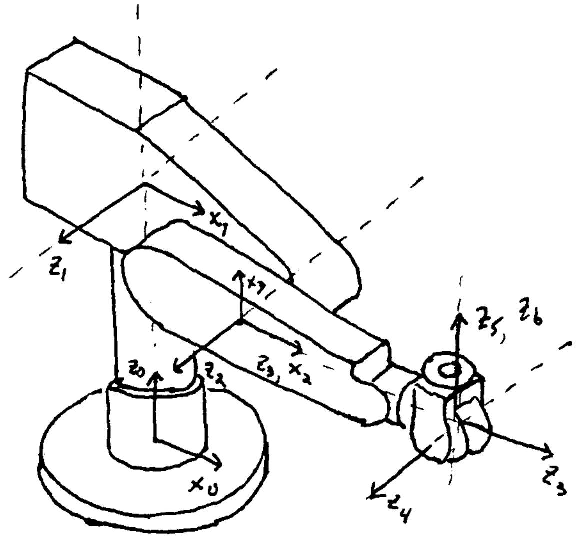
3–9 Attaching a spherical wrist to the robot of Problem 3-7 gives

$T_0^6 = T_0^3 T_3^6$

The matrix T_0^3 is given as in Problem 3-7. The matrix T_3^6 is given by Equation (3.3.11) of the text. Therefore

$$T_0^6 = \begin{bmatrix} -c_6s_5 & s_5s_6 & c_5 & d_3 + d_6c_5 \\ -c_4c_5c_6 + s_4s_6 & c_4c_5s_6 + c_6s_4 & -c_4s_5 & d_2 - d_6c_4s_5 \\ -c_4s_6 - c_5c_6s_4 & -c_4c_6 + c_5s_4s_6 & -s_4s_5 & d_1 - d_6s_4s_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3–10



link	a_i	α_i	d_i	θ_i
1	0	90°	13"	θ_1
2	8"	0	d_2	θ_2
3	8"	90°	0	θ_3
4	0	−90°	d_4	θ_4
5	0	90°	0	θ_5
6	0	0	d_6	θ_6

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 13 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 8c_2 \\ s_2 & c_2 & 0 & 8s_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} c_3 & 0 & s_3 & 8c_3 \\ s_3 & 0 & -c_3 & 8s_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

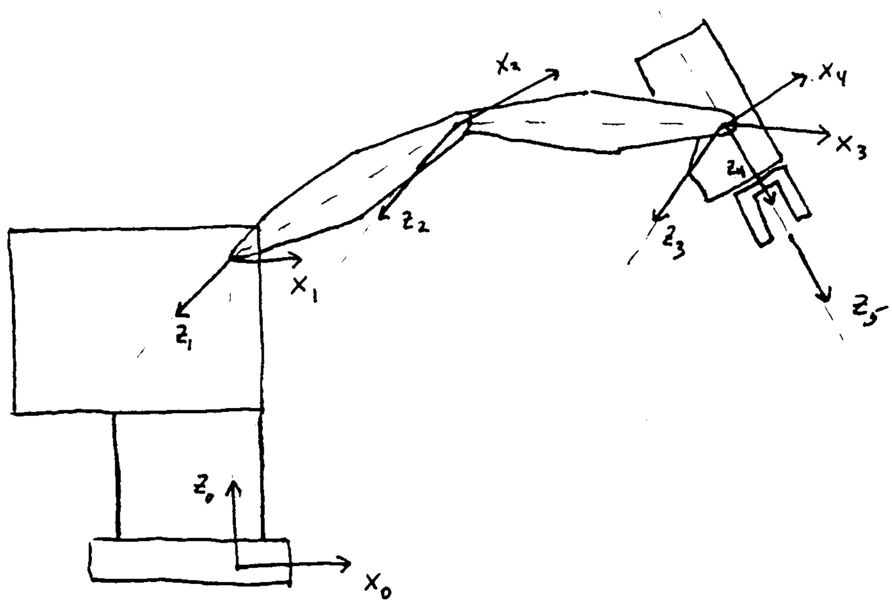
$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where:

$$r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_5c_6s_{23}] + s_1[c_4s_6 + s_4c_5c_6]$$
$$r_{12} = c_1[-c_{23}(c_4c_5s_6 + s_4c_6) + s_5s_6s_{23}] + s_1[c_4c_6 - s_4c_5s_6]$$
$$r_{13} = c_1[c_4s_5c_{23} + c_5s_{23}] - s_1s_4s_5$$
$$d_x = d_2s_1 + d_4c_1s_{23} + d_6[c_1(c_4s_5c_{23} + c_5s_{23}) + s_1s_4s_5] + 8c_1[c_{23} + c_2]$$
$$r_{21} = -c_1[c_4s_6 + s_4c_5c_6] + s_1[c_{23}(c_4c_5c_6 + s_4s_6) - s_5c_6s_{23}]$$
$$r_{22} = c_1[s_4c_5s_6 - c_4c_6] + s_1[-c_{23}(c_4c_5s_6 + s_4c_6) + s_5s_6s_{23}]$$
$$r_{23} = -c_1s_4s_5 + s_1[c_4s_5c_{23} + c_5s_{23}]$$
$$d_y = -d_2c_1 + d_4s_1s_{23} + d_6[s_1(c_4s_5c_{23} + c_5s_{23}) - c_1s_4s_5] + 8s_1[c_{23} + c_2]$$
$$r_{31} = s_{23}(c_4c_5c_6 - s_4s_6) + s_5c_6c_{23}$$
$$r_{32} = -s_{23}(c_4c_5s_6 + s_4c_6) - s_5s_6c_{23}$$
$$r_{33} = -c_5c_{23} + c_4s_5s_{23}$$
$$d_z = 13 - d_4c_{23} + d_6[-c_5c_{23} + c_4s_5s_{23}] + 8[s_{23} + s_2]$$

3–11



link	a_i	α_i	d_i	θ_i
1	0	90°	9.75	θ_1
2	9	0	0	θ_2
3	9		0	θ_3
4	0	90°	0	θ_4
5	0	0	5.125	θ_5

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 9.75 \\ 1 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 9c_2 \\ s_2 & c_2 & 0 & 9s_2 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & 9c_3 \\ s_3 & c_3 & 0 & 9s_3 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}; \quad A_5 = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & 5.125 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^5 = A_1\ A_2\ A_3\ A_4\ A_5 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$\begin{aligned}
r_{11} &= c_1c_5c_{234} - s_1s_5 \\
r_{12} &= -c_1s_5c_{234} + s_1c_5 \\
r_{13} &= c_1s_{234} \\
d_x &= 9c_1(c_2 + c_{23}) + 5.125c_1s_{234} \\
r_{12} &= s_1c_5c_{234} - c_1s_5 \\
r_{22} &= -s_1s_5c_{234} - c_1c_5 \\
r_{23} &= s_1s_{234} \\
d_y &= 9s_1(c_2 + c_{23}) + 5.125s_1s_{234} \\
r_{31} &= c_5s_{234} \\
r_{23} &= -s_5s_{234} \\
r_{33} &= -c_{234} \\
d_z &= 9.75 + 9[s_2 + s_{23}] - 5.125c_{234}
\end{aligned}$$

Note: In reality the Rhino Robot does not have a spherical wrist, since z_4 does not intersect z_3. There is an additional offset a_4 which should be added for complete accuracy. For simplicity we have not included this offset in this "academic example".

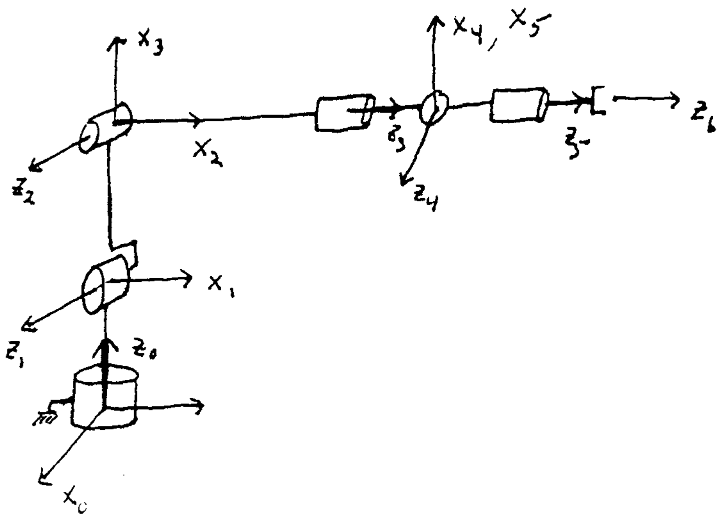
3–12

$$T_s^0 = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $T_s^5 = T_s^0 T_0^5$. Therefore

$$T_s^5 = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x + x \\ r_{21} & r_{22} & r_{23} & d_y + y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3–13



link	a_i	α_i	d_i	θ_i
1	0	90°	d_1	θ_1
2	0	0	d_2	θ_2
3	0	90°	0	θ_3
4	0	−90°	d_4	θ_4
5	0	90°	0	θ_5
6	0	0	d_6	θ_6

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$
\begin{aligned}
r_{11} &= c_1c_4c_5c_6c_{23} - c_1s_4s_6c_{12} - c_1c_6s_5s_{12} + s_1(c_4s_6 + s_4c_5c_6)\\
r_{12} &= -c_1c_4c_5c_6c_{23}c_{23} - c_1c_6s_4c_{23} + c_1s_5s_6s_{23} + s_1(c_4c_6 - c_5s_4s_6)\\
r_{13} &= c_1c_4s_5c_{23} + c_1c_5s_{23} + s_1s_4s_5\\
r_{21} &= c_4c_5c_6s_1c_{23} - s_1s_4s_6c_{23} - c_6s_1s_5s_{23} - c_1(c_4s_6 + c_5c_6s_4)\\
r_{22} &= -c_4c_5s_1s_6c_{23} + s_1s_5s_6s_{23} + c_1(c_5s_4s_6 - c_4c_6)\\
r_{23} &= c_4s_1s_5c_{23} + c_5s_1s_{23} - c_1s_4s_5\\
r_{31} &= c_4c_5c_6s_{23} - s_4s_6s_{23} + c_6s_5c_{12}\\
r_{32} &= -c_4c_5s_6s_{23} - s_5s_6c_{23} - c_6s_4s_{23}\\
r_{33} &= c_4s_5s_{23} - c_5c_{23}\\
d_x &= d_2s_1 + d_4c_1s_{23} + d_6(c_1c_4s_5c_{23} + c_1c_5s_{23} + s_1s_4s_5)\\
d_y &= -d_2c_1 + d_4s_1s_{23} + d_6(-c_4s_1s_5c_{23} + c_5s_1s_{23})\\
d_z &= d_1 - d_4c_{23} + d_6(c_4s_5s_{23} - c_5c_{23})
\end{aligned}
$$