2. det (X·X') = det (I) = 1 ;If X is orthogonal, det (X·X')=1

det (X·X') = det(X) · det(X')

determinant of a matrix is equal to the determinant of its transpose,

$\therefore$ det (X·X') = $(\det(X))^2 = 1$
$\therefore$ det (X) = $\pm$ 1

(3) $$R = \begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{bmatrix}$$

In a right handed co-ordinate system, $r_1 \times r_2 = r_3$ (r1: column1, r2: column2, r3: column3)

so, $r_{11}r_{22} - r_{12}r_{21} = r_{33}$
$r_{12}r_{23} - r_{13}r_{22} = r_{31}$
$r_{13}r_{21} - r_{11}r_{23} = r_{32}$

$$\begin{aligned} \det R &= r_{31}(r_{12}r_{23} - r_{13}r_{22}) + r_{32}(r_{13}r_{21} - r_{11}r_{23}) + r_{33}(r_{11}r_{22} - r_{12}r_{21}) \\ &= (r_{31} \times r_{31}) + (r_{32} \times r_{32}) + (r_{33} \times r_{33}) \\ &= \| r_{33} \| \quad = 1 \end{aligned}$$

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(4) **2.1.14**

$$R_{z,\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{z,0} = \begin{bmatrix} \cos 0 & \sin 0 & 0 \\ \sin 0 & \cos 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

2.1.15:

$$R_{z,\theta} \cdot R_{z,\phi} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta\cdot\cos\phi + \sin\theta\cdot\sin\phi & \cos\theta\cdot\sin\phi + \cos\phi\cdot\sin\theta & 0 \\ \cos\phi\cdot\sin\theta + \cos\theta\cdot\sin\phi & \cos\theta\cdot\cos\phi + \sin\theta\cdot\sin\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+\phi) & \sin(\theta+\phi) & 0 \\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= R_{z,\theta+\phi}$$

2.1.16:

$$R_{z,\theta}^{-1} = R_{z,-\theta}$$

$$R_{z,\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

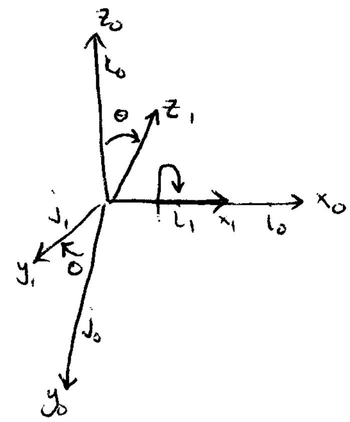
$$\text{adj } R_{z,\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}' = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{z,\theta}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = R_{z,-\theta}$$

(5) Derive **2.1.17** and **2.1.18**

<u>2.1.17</u>:

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

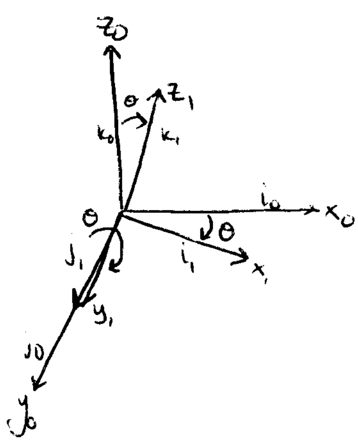


$$R^1_0 = \begin{bmatrix} i_1 i_0 & j_1 i_0 & k_1 i_0 \\ i_1 j_0 & j_1 j_0 & k_1 j_0 \\ i_1 k_0 & j_1 k_0 & k_1 k_0 \end{bmatrix}$$

$i_1 i_0 = 1 \quad (\theta = 0)$
$j_1 j_0 = \cos\theta$
$k_1 j_0 = -\sin\theta$
$j_1 k_0 = \sin\theta$
$k_1 k_0 = \cos\theta$
The rest of the terms are zero. Hence shown

<u>2.1.18</u>:

$$R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$



$$R^1_0 = \begin{bmatrix} i_1 i_0 & j_1 i_0 & k_1 i_0 \\ i_1 j_0 & j_1 j_0 & k_1 j_0 \\ i_1 k_0 & j_1 k_0 & k_1 k_0 \end{bmatrix}$$

$j_1 j_0 = 1 \quad (\theta = 0 ; \cos\theta = 1)$
$i_1 i_0 = \cos\theta$
$k_1 i_0 = \sin\theta$
$i_1 k_0 = -\sin\theta$
$k_1 k_0 = \cos\theta$
The rest of the terms are zero

$$\therefore \quad R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

(6) Let $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} a_4 & -a_2 \\ -a_3 & a_1 \end{bmatrix} = \begin{bmatrix} a_4 & -a_2 \\ -a_3 & a_1 \end{bmatrix} \qquad (\because \det(A) = 1)$$

$$A^T = \begin{bmatrix} a_1 & a_3 \\ a_2 & a_4 \end{bmatrix}$$

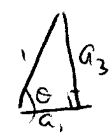
$A^T = A^{-1}$

$\therefore \left.\begin{aligned} a_1 &= a_4 \\ a_2 &= -a_3 \end{aligned}\right\}$ by comparison

$\det(A) = a_1^2 + a_3^2$

Given $\det(A) = 1 = a_1^2 + a_3^2$

$\therefore$ a_1 and a_3 are at 90° to each other



$\cos\theta = a_1$
$\sin\theta = a_3$

$$A = \begin{bmatrix} a_1 & -a_3 \\ a_3 & a_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\rightarrow (R_{y,\pi/2} \cdot R_{x,\pi/4} \cdot R_{z,\pi/2})$

(7) Rotation matrix $= R_{z,\pi/4} \cdot R_{x,\pi/2} \cdot R_{y,\pi/2}$

$$= \begin{bmatrix} \cos\pi/4 & -\sin\pi/4 & 0 \\ \sin\pi/4 & \cos\pi/4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\pi/2 & -\sin\pi/2 \\ 0 & \sin\pi/2 & \cos\pi/2 \end{bmatrix} \cdot \begin{bmatrix} \cos\pi/2 & 0 & \sin\pi/2 \\ 0 & 1 & 0 \\ -\sin\pi/2 & 0 & \cos\pi/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

-5

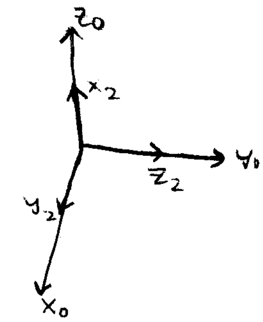
$$= \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix} \#$$

(8) Rotation matrix $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix}_x \cdot \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}_y$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



(9) $R^3_1 = R^2_1 \cdot R^3_2$

$\Rightarrow \quad R^3_2 = (R^2_1)^{-1} \cdot R^3_1$

$$R^2_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix}$$

$$(R^2_1)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & +\sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 1/2 \end{bmatrix}$$

$$R^3_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ \sqrt{3}/2 & 1/2 & 0 \\ 1/2 & -\sqrt{3}/2 & 0 \end{bmatrix}$$

#

(12) $k = 1/\sqrt{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$

$\theta = 90°$

$$R_{k,\theta} = \begin{bmatrix} k_x^2 v_\theta + c\theta & k_x k_y v_\theta - k_z s\theta & k_x k_z v_\theta + k_y s\theta \\ k_x k_y v\theta + k_z s\theta & k_y^2 v\theta + c\theta & k_y k_z v\theta - k_x s\theta \\ k_x k_z v\theta - k_y s\theta & k_y k_z v\theta + k_x s\theta & k_z^2 v\theta + c\theta \end{bmatrix}$$

$v\theta = 1 - c\theta$
$\cos\theta = c\theta$
$\sin\theta = s\theta$

$c90° = 0$
$v90° = 1$
$s90° = 1$

$$= \begin{bmatrix} 1/3 & 1/3 - 1/\sqrt{3} & 1/3 + 1/\sqrt{3} \\ 1/3 + 1/\sqrt{3} & 1/3 & 1/3 - 1/\sqrt{3} \\ 1/3 - 1/\sqrt{3} & 1/3 + 1/\sqrt{3} & 1/3 \end{bmatrix}$$

(14) 90° about y_0

45^0 about z_1

$\therefore R = R_{x_0,45^\circ} \cdot R_{z_0,90^\circ}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$Tr(R) = 1/\sqrt{2}$

$$\theta = \cos^{-1}\left(\frac{Tr(R)-1}{2}\right) = \cos^{-1}\left(\frac{-1/\sqrt{2}}{2}\right) = \cos^{-1}\left(\frac{-1}{2\sqrt{2}}\right) = 110.71^\circ$$

$$k = \frac{1}{2\sin(110.71^\circ)}\begin{bmatrix} \sqrt{2} \\ -1/\sqrt{2} \\ \frac{\sqrt{2}+1}{2} \end{bmatrix} = \frac{1}{2(0.935)}\begin{bmatrix} 1.414 \\ -0.707 \\ 1.707 \end{bmatrix} = 0.535\begin{bmatrix} 1.414 \\ -0.707 \\ 1.707 \end{bmatrix} = \begin{bmatrix} 0.756 \\ -0.378 \\ 0.913 \end{bmatrix}$$

(15) Euler angles $\{\pi/2, 0, \pi/4\}$

$R_0^1 = R_{z,0} \cdot R_{y,\pi/2} \cdot R_{z,\pi/4}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

#

(16) $H_0^1 = \text{Trans}_{x,3} \cdot R_{z,\pi/2} \cdot \text{Trans}_{y,1}$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\pi/2 & -\sin\pi/2 & 0 & 0 \\ \sin\pi/2 & \cos\pi/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#

(17) $H_0^1 = R_{y_0,-90^\circ} \cdot R_{z_0,-90^\circ} \cdot \text{Trans}_{z_0,1m}$

$H_0^2 = R_{x_0,-90^\circ} \cdot R_{z_0,-90^\circ} \cdot \text{Trans}_{y_0,1m}$

$H_1^2 = R_{x_0,-90^\circ} \cdot R_{y_0,90^\circ} \cdot \text{Trans}_{z_0,-1m} \cdot \text{Trans}_{y_0,1m}$

[illegible]

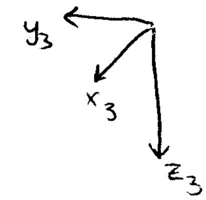
(18) $H_0^1 = \text{Trans}_{y_0,1m} \cdot \text{Trans}_{z_0,1m}$

$H_0^2 = \text{Trans}_{y_0,1.5m} \cdot \text{Trans}_{x_0,-0.5m} \cdot \text{Trans}_{z_0,1m}$

$H_0^3 = \text{Trans}_{y_0,1.5m} \cdot \text{Trans}_{x_0,-0.5m} \cdot \text{Trans}_{z_0,3m} \cdot R_{x_0,-180^\circ} \cdot R_{z_0,90^\circ}$

$H_3^2 = \text{Trans}_{z_0,-2m} \cdot R_{x_0,180^\circ} \cdot R_{z_0,90^\circ}$

(19) $R_0^3 = \text{Trans}_{y_0, 1.5m} \cdot \text{Trans}_{x_0, -0.5m} \cdot \text{Trans}_{z_0, 3m} \cdot R_{x_0, 180^\circ}$



$R_3^2 = \text{Trans}_{z_0, -2m} \cdot R_{x_0, 180^\circ}$

(20) Block frame relative base frame:

$R_0^{block} = \text{Trans}_{y_0, 1.8} \cdot \text{Trans}_{z_0, 1.1} \cdot R_{z_0, 90^\circ}$

Block frame to camera frame:

$R_3^{block} = \text{Trans}_{z_3, 1.9m} \cdot \text{Trans}_{y_3, 0.5m} \cdot R_{z_3, -90^\circ} \cdot R_{x_3, 180^\circ}$.

— x —