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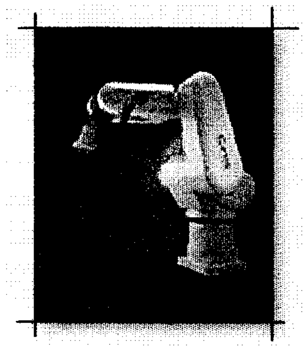
Robotics, CpE 360



Assignment # 2

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Q. # 2.3 Show that Det R = +1, if we restrict ourselves to right - handed coordinate systems.
Solution:

Let us consider the following matrix:

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M = {{R11, R12, R13}, {R21, R22, R23}, {R31, R32, R33}}; MatrixForm[M]
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$$\begin{pmatrix} R11 & R12 & R13 \\ R21 & R22 & R23 \\ R31 & R32 & R33 \end{pmatrix}$$

But we know that, right - handed corordinate system implies: R1 * R2 = R3

When expanding the above matix A through column 3, we get

$$\text{DetA} = \text{Det}\left[\begin{pmatrix} R11 & R12 & R13 \\ R21 & R22 & R23 \\ R31 & R32 & R33 \end{pmatrix}\right]$$

$$R13\,(-R13\,R22 + R12\,R23) - (-R13\,R21 + R11\,R23)\,R32 + (-R12\,R21 + R11\,R22)\,R33$$

$$R31^2 + R32^2 + R33^2$$

$|| R3 ||^2$

1

Thus Det R = +1 has been observed when we consider the right - handed coordinate systems .

Q . #2 .4 Verify Equations 2.1 .14 - 2.1 .16
Solution :

Equation 2.1 .14 can be expressed as :

R (z, 0) = I
Since, R (z, 0)

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Imat = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}; MatrixForm[Imat]
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$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It is obvious that R (z, 0) = I .

Equation 2.1 .15 can be expressed as :

$$R(z, \theta) * R(z, \phi) = R(z, \theta + \phi)$$

we know that,

$$\begin{pmatrix} R11 & R12 & R13 \\ R21 & R22 & R23 \\ R31 & R32 & R33 \end{pmatrix}$$

$$R(z, \theta) = \begin{pmatrix} \text{Cos}\theta & -\text{Sin}\theta & 0 \\ \text{Sin}\theta & \text{Cos}\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(z, \phi) = \begin{pmatrix} \text{Cos}\phi & -\text{Sin}\phi & 0 \\ \text{Sin}\phi & \text{Cos}\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(z, \theta) * R(z, \phi) = \begin{pmatrix} \text{Cos}\theta & -\text{Sin}\theta & 0 \\ \text{Sin}\theta & \text{Cos}\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{Cos}\phi & -\text{Sin}\phi & 0 \\ \text{Sin}\phi & \text{Cos}\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}(\phi + \theta) & -\text{Sin}(\theta + \phi) & 0 \\ \text{Sin}(\phi + \theta) & \text{Cos}(\theta + \phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Equation 2.1 .16 can be expressed as :

$$R(z, \theta)^{-1} = R(z, -\theta)$$

we know that,

$R(z, \theta)\, R(z, -\theta) = R(z, \theta - \theta) = I$

Q. # 2.6 Given that,

A is a 2 x2 roataion matrix.

$A^T A = I$

$Det A = 1$

so, $A = \begin{pmatrix} Cos\theta & -Sin\theta \\ Sin\theta & Cos\theta \end{pmatrix}$

Solution :

Let, $A = \begin{pmatrix} x & y \\ w & z \end{pmatrix}$

Using Cramer's rule, we get :

$$A^{-1} = \begin{pmatrix} x & -y \\ -w & z \end{pmatrix}$$

such, which

$x = y$; **and** $y = -w$;

So,

$$A = \begin{pmatrix} x & -y \\ -w & z \end{pmatrix}$$

$$Det A = x^2 + y^2 = 1$$

Now, it can be said that there exists a unique θ such that A can be expressed as above. That is :

$\theta = \tan^{-1}[w / x]$

This yields, $Cos\theta = x$ **and** $w = Sin\theta$.

Thus,

$A = \begin{pmatrix} \text{Cos}\theta & -\text{Sin}\theta \\ \text{Sin}\theta & \text{Cos}\theta \end{pmatrix}$ **happens to be true .**

Q . #2 .7 Find the rotation matrix representing a roll of π / 4 followed by a law of π / 2 followed by a pitch of π / 2
Soltuon :

We know that :

$$R(0,1) = R(y, \pi/2)\, R(x, \pi/4)\, R(z, \pi/2)$$

$$\begin{pmatrix} \text{Cos}\,\pi/2 & 0 & \text{Sin}\,\pi/2 \\ 0 & 1 & 0 \\ -\text{Sin}\,\pi/2 & 0 & \text{Cos}\,\pi/2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}\,\pi/4 & -\text{Sin}\,\pi/4 \\ 0 & \text{Sin}\,\pi/4 & \text{Cos}\,\pi/4 \end{pmatrix} \begin{pmatrix} \text{Cos}\,\pi/2 & -\text{Sin}\,\pi/2 & 0 \\ \text{Sin}\,\pi/2 & \text{Cos}\,\pi/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Multiplying the above matrices gives a result of

$$\begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \end{pmatrix}$$

which is the required rotation matrix

Q . #2 .8 If the coordinate frame 01 x1y1z1 is obtained from the coordinate frame o0x0y0z0 by a rotation of π / 2 about the x - axis followed by a rotation of π / 2 about the fixed y - axis, find the rotation matrix R representing the composite transformation .
Solution :

We know that :

$$R(2,0) = R(y, \pi/2)\, R(x, \pi/2)$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

The final result is :

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$$

Q . #2 .9 Suppose that the three coordinate frames frames 01 x1y1z02x2y2z2 and 03 x3y3z3 are given, and suppose

$$R(2,1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{pmatrix};\; R(3,1) = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Find the matrix R (3, 2)
Solution :

We know

$$R(3, 2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

The above product is

$$\begin{pmatrix} 0 & 0 & -1 \\ \sqrt{3}/2 & 1/2 & 0 \\ 1/2 & -\sqrt{3}/2 & 0 \end{pmatrix}$$

Q. #2.12 let $k = 1/\sqrt{3}\ (1, 1, 1)\ T,\ \theta = 90°$. *Find* $R(k, \theta)$
Solution :

$$R(k, \theta) = \begin{pmatrix} 1/3 & 1/3 - 1/\sqrt{3} & 1/3 + 1/\sqrt{3} \\ 1/3 + 1/\sqrt{3} & 1/3 & 1/3 - 1/\sqrt{3} \\ 1/3 - 1/\sqrt{3} & 1/3 + 1/\sqrt{3} & 1/3 \end{pmatrix}$$

Q. #2.14 Suppose R represents a rotation of 90 ° about y0 followed by a rotation of 45 ° about z1. Find the equivalent axis / angle to represent R. Sketch the initial and final frames and the equivalent axis vector k.
Solution :

$$R = R(y, 90)\,R(z, 45) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \end{pmatrix}$$

$$K = 1/\,2\sin\theta \begin{pmatrix} r32 - r23 \\ r13 - r31 \\ r21 - r12 \end{pmatrix}$$

Resulting in

$$(0.5054481) \begin{pmatrix} 0.7071068 \\ 1.70701068 \\ 0.7071068 \end{pmatrix}$$

Q. #2.15 Find the rotation matrix corresponding to the set Euler angles $(\pi/2,\ 0,\ \pi/4)$. What is the direction of the x1 axis relative to the base frame?
Solution :

We know that :

$$R(0,1) = \begin{pmatrix} 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix}$$

Hence it can be expressed as the direction of the x - axis is

$i = \left(0,\ 1/\sqrt{2},\ -1/\sqrt{2}\right) T.$

Q. #2.16 Compute the homogenous transformation reperesnting a translation of 3 units along the x - axis followed by a rotation of $\pi/2$ about the current z - axis followed by translation of 1 unit along the the y - axis. Sketch the frame. what are the coordinates of the origin 01 with respect to the original frame in each case?
Solution :

We have,

$T = T(y,\ 1)\ T(x,\ 3 - z,\ \pi/2)$

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Imat2 = {{1, 0, 0, 0}, {0, 1, 0, 1}, {0, 0, 1, 0}, {0, 0, 0, 1}}; MatrixForm[Imat2]
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$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Resulting in

$$\begin{pmatrix} 1 & -1 & 0 & 3 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Q. #2.17 Consider the diagram of Figure 2 - 10. Find the homogeneous transformation $H(1,\ 0)$, $H(2,\ 0)$, $H(2,\ 1)$ representing the transformations among the three frames shown. Show that $H(2,\ 0) = H(1,\ 0)\ H(2,\ 1)$.
Solution :

We know that :

$$H(1, 0) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H(2, 0) = \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H(2, 1) = \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H(2, 0) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Q. #2.18 Find the homogeneous transformation relating each of the frames to the base frame 00 x0y0z0. Find the homogeneous transformation relating to the frame 02 x2y2z2 to the camera frame 03 x3y3z3.
Solution:

We can expressed the followings as :

$$H(1, 0) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H(2, 0) = \begin{pmatrix} 1 & 0 & 0 & -.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H(3, 0) = \begin{pmatrix} 0 & 1 & 0 & -.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 1.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H(3, 2) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Q. #2.19 In problem 2.18 suppose that, after the camera is calibrated,

it is rotated 90 ° about the axis z3 Recompute the above coordinate transformations.
Solution :

We know

$$H(2,1) = \begin{pmatrix} 1 & 0 & 0 & -.5 \\ 0 & 1 & 0 & .5 \\ 0 & 0 & 1 & .1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H(2,0) = H(1,0)\, H(2,1)$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -.5 \\ 0 & 1 & 0 & .5 \\ 0 & 0 & 1 & .1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & -.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Q. #2.20 Compute the homogeneous transformation relating the block frame to the camera frame; the block frame to the base frame.
Solution :

$$H(3,2) = \begin{pmatrix} 1 & 0 & 0 & -.3 \\ 0 & 1 & 0 & .4 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The homogeneous transformation can be expressed as :

$$H(2,0) = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & .8 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
