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Robotics, Homework #1

1-1 Compared to other automated machines, for example CNC milling machines ,robots are more flexible, than are specialized machines, since robots were born as a combination of two technologies : mechanical linked machines and programmable machines, such as CNC. The ultimate robot is the one, which can "think".

1-2 **Forward kinematics**- determination of the position and orientation of the end-effector, when given joint variables.

Inverse kinematics - solving the joint angles, when given the position of end-effector.

Trajectory planning – determining a control inputs for a manipulator to follow a desired trajectory.

Workspace- total volume swept out by the end-effector as the manipulator executes all possible motions, which is constrained by the geometry of manipulator and constraints of the joints.

Accuracy- is a measure of how close the manipulator can come to a given point within its work space

Repeatability - is a measure of how close a manipulator can return to a previously taught point.

Resolution - the smallest increment of motion of the link, that controller can sense .

Joint Variable - represents a relative displacement between adjacent links.

Spherical wrist - kinematic chain between the arm and the hand of the robot, whose joint axes intersect at a common point .

End-effector - the hand of the robot, the actual work performer.

1-3 Robots are classified by their **power source**(electrically, hidraulically, or pneumatically powered) , **application area**(assembly, non-assembly) , and **method of control**(servo controlled robots or non-servo, open-loop, robots).

1-4 Non-servo robots, which are simple open loop robots are mainly useful for a material transfer. Point-to-Point robots are limited in their use , they can be useful, for example in making a predefined number of holes in materials, since they are programmed to move between discrete number of points. Continuous path robots are widely used , for example in welding or chip manufacturing .

1-5 Continuos path robots can do welding, which p-to-point robots can't do, or cutting according along the predefined line, or polishing of a materials, or plotting the complex lines, or car painting (if point-to-point robot would paint a car customers would complain).

1-6 A computer vision in robotics would be useful in the edge detection, or for a moving robot avoiding obstacles is also important, it would be useful in color detection in painting, in automatic detection of the intensity of the light source (robots, that can be used in hazardous areas) and so on.

1-7 If every factory in US will be reopened tomorrow fully automated, there could be unpredictable economic and social consequences, let's say a millions of people would be replaced by machines, while production may improve, the social tension would be inevitable, which could lead to a mass disturbances and revolution. Besides not many people have been trained to maintain and program robots, some factories may experience a shortage of qualified personnel . I think, that every technical innovation should be implemented gradually, bringing prosperity to people, instead of suffering.

1-8 The law , that would ban all future use of industrial robots in US, will significantly slow down economic and social progress, this law would even destroy some major technologies, such as modern car, aviation or computing equipment production, since they it would be impossible in our days without automation.

1-9 Redundant manipulators would be useful, for example, in material transfer , let's say it is necessary to finish a transfer faster, than several loads could be taken at the same time, or for a heavy loads, let's suppose, that a load's weight exceeds max capacity of a manipulator, then a redundant manipulator can be handy to support the other manipulator, just like human hands.

```
//  Timur Nurullaev, Robotics
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//
//   Program calculates simple two planar robot kinematics.
//
//   Input of angle values in degrees, not in radians.


#include<iostream.h>
#include<math.h>


void main()
{
   float x, y, q1, q2, a1, a2,D;
   float x1, y1, q11, q21, x2, y2, q12, q22;
   float a, b, c, d, e, f, g, h;

   cout << "q1(DEG.)=?";
   cin >> q1;
   cout << "q2(DEG.)=?";
   cin >> q2;
   cout << "a1=?";
   cin >> a1;
   cout << "a2=?";
   cin >> a2;

   x=a1*cos(M_PI/180*q1)+a2*cos(M_PI/180*(q1+q2));
   y=a1*sin(M_PI/180*q1)+a2*sin(M_PI/180*(q1+q2));
   D = (x*x +y*y-a1*a1-a2*a2)/(2*a1*a2);
   cout << "x = " <<x<< endl;
   cout << "y = " <<y<< endl;
//---------------------------------------------------------
//   Inverse Kinematics, use x and y as an output of the
//   previous solution
//---------------------------------------------------------
   cout << "q1= "

<<(atan(y/x)-
(atan(a2*sin(M_PI/180*q2)/(a1+a2*cos(M_PI/180*q2)))))*180/M_PI
<<"(DEG.)"<<endl;
   cout << "q2= " <<acos(D)*180/M_PI<<"(DEG.)"<<endl;

//----------------------------------------------------------------
//         Velocity kinematics, q1 and q2 are outputs from previous
//                   solution
//----------------------------------------------------------------

   cout <<"------------------------------------------------------------" <<
endl;
   cout <<"joint velocity q11=? ";
   cin >> q11;
   cout <<"joint velocity q12=? ";
   cin >> q12;

   x1 = -a1*sin(M_PI/180*q1)*q11- (a2*sin(M_PI/180*(q1+q2))*(q11+q12)) ;
   y1 = a1*cos(M_PI/180*q1)*q11+ (a2*cos(M_PI/180*(q1+q2))*(q11+q12));

   cout <<" end effector velocity : x'="<<x1<<"  y'="<<y1<< endl;
```
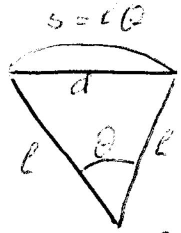

```
  q11 = (a2*cos(M_PI/180*(q1+q2))*x1 +a2*sin(M_PI/180*(q1+q2))*y1)/
         (a1*a2*sin(M_PI/180*q2));

 q12 = (  x1 * (-a1 * cos(M_PI/180*q1)   -   a2 * cos(M_PI/180*(q1+q2))
) -
         y1 *   ( a2 * sin(M_PI/180*(q1+q2))   +  a1 * sin(M_PI/180*q1) )
)  /
           (a1*a2*sin(M_PI/180*q2));


   cout <<"joint velocity q1' =" <<q11 << endl;
   cout <<"joint velocity q2' =" <<q12 << endl;

//-------------------------------------------------------------------
//   Acceleration kinematics
//-------------------------------------------------------------------

 cout <<"------------------------------------------------------------" <<
endl;
   cout <<"joint acceleration q1''=? ";
   cin >> q21;
   cout <<"joint acceleration q2''=? ";
   cin >> q22;

 x2 = -a1 * ( cos(M_PI/180*q1)* q11*q11 + sin(M_PI/180*q1) * q21) -
        a2 * ( cos(M_PI/180*(q1+q2))*(q11+q12)*(q11+q12) +
               sin(M_PI/180*(q1+q2))*(q21+q22) );

  y2 = a1 * ( -sin(M_PI/180*q1)*q11*q11 + cos(M_PI/180*q1)* q21*q21 ) +
       a2 * ( -sin(M_PI/180*(q1+q2)) * (q11+q12)*(q11+q12) +
               cos(M_PI/180*(q1+q2)) *  (q21+q22) );

  cout <<" end effector acceleration : x''="<<x2<<"  y''="<<y2<< ehdl;


q21 = (     ( a2*x1*cos(M_PI/180*(q1+q2)) + a2*y1*sin(M_PI/180*(q1+q2)) )
*
             a1*a2*a2*cos(M_PI/180*q2)*q12    -  a1*a2*sin(M_PI/180*q2)
*
           ( -sin(M_PI/180*(q1+q2))*(q11+q12)*x1 +
cos(M_PI/180*(q1+q2))*x2
             +cos(M_PI/180*(q1+q2))*(q11+q12)*y1 +
sin(M_PI/180*(q1+q2))*y2 )   )
          /(a1*a1*a2*a2*sin(M_PI/180*q2)*sin(M_PI/180*q2));


 a = -a1*( -sin(M_PI/180*q1)*q11*x1 + cos(M_PI/180*q1)*(q11*y1+x2) +
           sin(M_PI/180*q1)*y2 );

 b = a2*( -sin(M_PI/180*(q1+q2)) * (q11+q12)* x1 + cos(M_PI/180*(q1+q2))
* x2 +
          cos(M_PI/180*(q1+q2)) * (q11+q12)* y1 + sin(M_PI/180*(q1+q2))
*y2 );

 c = (a-b)*a1*a2*sin(M_PI/180*q2);

 d = x1*( -a1*cos(M_PI/180*q1)-a2*cos(M_PI/180*(q1+q2)) ) -
     y1*( a1*sin(M_PI/180*q1)+a2*sin(M_PI/180*(q1+q2))   );
```

```
 e = a1*a2*cos(M_PI/180*q2)*q12 * d;

 h = c-e;


q22 = h / (a1*a1*a2*a2*sin(M_PI/180*q2)*sin(M_PI/180*q2) );


   cout <<"joint acceleration q1'' =" <<q21 << endl;
   cout <<"joint acceleration q2'' =" <<q22 << endl;

}
```

1-14



law of cosines:

$d^2 = l^2 + l^2 - 2l^2 \cdot \cos\theta = 2l^2 - 2l^2 \cdot \cos\theta$

$\Rightarrow d = l\sqrt{2(1-\cos\theta)}$

• Resolution of the linear link is:

$\frac{d}{2^{10}} = \frac{1 \cdot \sqrt{2(1-\cos(\frac{\pi}{2}))}}{2^{10}} = \frac{1 \cdot \sqrt{2(1-0)}}{2^{10}} = \frac{1.414}{2^{10}} =$

$= (0.00138\ m)$

• Resolution of rotational link: $\frac{s}{2^{10}} = \frac{\pi}{2} = \frac{3.1415}{2}$

$= 1.570\ m = (0.001533\ m)$

Check page 3 answer key

✓ V. good

1-15 $s = 0.5m \cdot \pi = 0.5 \cdot 3.14 = 1.5707\ (m)$

Resolution of 8-bit encoder $\frac{1.5707}{2^8} = \frac{1.5707}{256} =$

$= 0.006135\ (m)$

✓

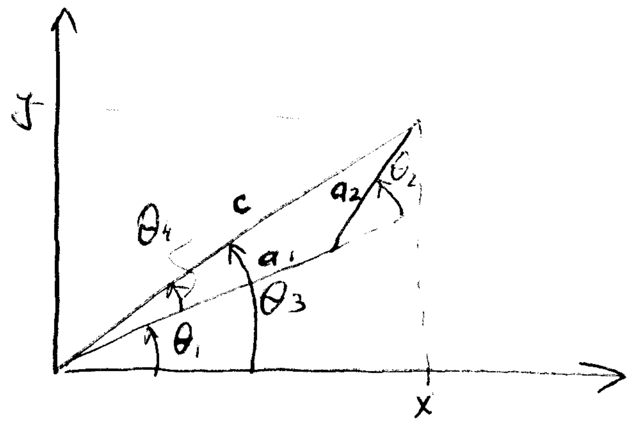
1-16 s of the motor shaft $= 1.5707 \cdot 50 = 78.535\ m$

$\Rightarrow$ resolution $= 0.006135 \cdot 50 = 0.306\ m$

1-17 Since repeatability is affected primarily by the control resolution (all the errors were taken into account and proper encoder values were set), the repeatability is higher than the accuracy (more accurate)

1-18 With the use of direct end-point sensing the manipulator accuracy would be improved, since machine accuracy wouldn't affect the end-effector position. However, using end-point direct sensing would introduce necessity to use sophisticated equipment and complex algorithms, which would require super-fast machines.

1-19



$$\theta_1 = \theta_3 - \theta_4$$

$$\theta_3 = a\cos\left(\frac{x}{c}\right)$$

$$\theta_4 = a\cos\left(\frac{a_1^2 - a_2^2 + c}{2a_1 \cdot c}\right)$$

according to the law of cosines

$$\theta_1 = a\cos\left(\frac{x}{c}\right) - a\cos\left(\frac{a_1^2 - a_2^2 + c}{2a_1 \cdot c}\right)$$

1-22

① $\dot{x} = -\sin\theta_1 - 3\sin(\theta_1 + \theta_2)$

$\dot{y} = \cos\theta_1 + 3\cos(\theta_1 + \theta_2)$

② $\dot{x} = -\sin\frac{\pi}{4} - 3\sin\left(\frac{\pi}{2}\right) = -0.707 - 3 = -3.707$

$\dot{y} = \cos\frac{\pi}{4} + 3\cos\left(\frac{\pi}{2}\right) = 0.707$

✓ good

Derivation of Inverse Jacobian in Inverse Kinematics

If $J = \begin{bmatrix} -a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2) & -a_2 \sin(\theta_1+\theta_2) \\ a_2 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) & a_2 \cos(\theta_1+\theta_2) \end{bmatrix}$

$$\Rightarrow J^{-1} = \begin{bmatrix} a_2 \cos(\theta_1+\theta_2) & a_2 \sin(\theta_1+\theta_2) \\ -(a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) & -a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2) \end{bmatrix}$$

$$\times \frac{1}{((-a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2)) \cdot a_2 \cos(\theta_1+\theta_2) + }$$

$$\frac{1}{+ \quad ((a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2)) \cdot a_2 \sin(\theta_1+\theta_2)}$$

- Simplifying the determinant we get:

<u>det</u> $= -a_1 a_2 \sin\theta_1 \cdot \cos(\theta_1+\theta_2) - a_2^2 \sin(\theta_1+\theta_2) \cdot \cos(\theta_1+\theta_2)$
$+ a_1 a_2 \sin(\theta_1+\theta_2) \cdot \cos\theta_1 + a_2^2 \cdot \cos(\theta_1+\theta_2) \sin(\theta_1+\theta_2) =$

$= a_1 a_2 (\underbrace{\sin(\theta_1+\theta_2) \cdot \cos\theta_1 - \sin\theta_1 \cdot \cos(\theta_1+\theta_2)})$

$\sin(\theta_1+\theta_2 - \theta_1)$ according $\sin(a-b) = \sin a \cdot \cos b - \sin b \cdot \cos a$

$= \boxed{a_1 a_2 \cdot \sin\theta_2} \rightarrow$ determinant

Velocity kinematics.

$$\dot{x} = -a_1 \sin\theta_1 \cdot \dot{\theta}_1 - a_2 \sin(\theta_1+\theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$
$$\dot{y} = a_1 \cos\theta_1 \cdot \dot{\theta}_1 + a_2 \cos(\theta_1+\theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$

if $x = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$ $\Rightarrow$

$$\dot{x} = \begin{bmatrix} -a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2) & -a_2 \sin(\theta_1+\theta_2) \\ a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) & a_2 \cos(\theta_1+\theta_2) \end{bmatrix} \cdot \dot{\theta} =$$

$$= J\dot{\theta} \Rightarrow \theta' = J^{-1} x' \Rightarrow$$

$$\begin{bmatrix} \theta_1' \\ \theta_2' \end{bmatrix} = \frac{1}{a_1 a_2 \sin\theta_2} \begin{bmatrix} a_2 \cos(\theta_1+\theta_2) & a_2 \sin(\theta_1+\theta_2) \\ -a_1 \cos\theta_1 - a_2 \cos(\theta_1+\theta_2) & -a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2) \end{bmatrix}$$
$$\cdot \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\Rightarrow \theta_1' = \frac{1}{a_1 a_2 \sin\theta_2}\left(a_2 \cos(\theta_1+\theta_2) \cdot x' + a_2 \sin(\theta_1+\theta_2) \cdot y'\right)$$

$$\theta_2' = \frac{1}{a_1 a_2 \sin\theta_2}\Big(\big(-a_1 \cos\theta_1 - a_2 \cos(\theta_1+\theta_2)\big)x' - y'(a_1 \sin\theta_1 +$$
$$+ a_2 \sin(\theta_1+\theta_2))\Big)$$

Acceleration kinematics.

$$y'' = a_1(-\sin\theta_1 \cdot (\theta_1')^2 + \cos\theta_1 \cdot \theta_1'') + a_2 \cdot (-\sin(\theta_1+\theta_2)(\theta_1'+\theta_2')^2 +$$
$$+ \cos(\theta_1+\theta_2) \cdot (\theta_1''+\theta_2''))$$

$$x'' = -a_1\left[\cos\theta_1 \cdot (\theta_1')^2 + \sin\theta_1 \cdot \theta_1''\right] - a_2\left[\cos(\theta_1+\theta_2)(\theta_1'+\theta_2')^2 + \sin(\theta_1+\theta_2)(\theta_1''+\theta_2'')\right]$$

$$\theta_1'' = \frac{[a_2\cos(\theta_1+\theta_2)\cdot x' + a_2\sin(\theta_1+\theta_2)\cdot y'] \cdot a_1 a_2 \cdot \cos\theta_2 \cdot \theta_2' \; -}{(a_1 a_2 \sin\theta_2)^2}$$

$$\frac{- a_1 a_2 \sin\theta_2 \cdot [a_2(-\sin(\theta_1+\theta_2)(\theta_1'+\theta_2')\cdot x' + \cos(\theta_1+\theta_2)\cdot x'' + \cos(\theta_1+\theta_2) \times}{(a_1 a_2 \sin\theta_2)^2}$$

$$\frac{\times (\theta_1'+\theta_2')\cdot y' + \sin(\theta_1+\theta_2)\cdot y'')]}{(a_1 a_2 \sin\theta_2)^2}$$

$$\theta_2'' = \frac{[-a_1(-\sin\theta_1 \cdot \theta_1' \cdot x' + \cos\theta_1 \, x'' + \cos\theta_1 \cdot \theta_1' \cdot y' + \sin\theta_1 \cdot y'') \; -}{(a_1 a_2 \sin\theta_2)^2}$$

$$\frac{- a_2(-\sin(\theta_1+\theta_2)(\theta_1'+\theta_2')\cdot x' + \cos(\theta_1+\theta_2)\cdot x'' + \cos(\theta_1+\theta_2) \times}{(a_1 a_2 \sin\theta_2)^2}$$

$$\frac{\times (\theta_1'+\theta_2)\cdot y' + \sin(\theta_1+\theta_2) y'')] \cdot a_1 a_2 \sin\theta_2 \; -}{(a_1 a_2 \sin\theta_2)^2}$$

$$\frac{- a_1 \cdot a_2 \cdot \cos\theta_2 \cdot \theta_2' [x' \cdot (-a_1\cos\theta_1 - a_2\cos(\theta_1+\theta_2)) - y'(a_1\sin\theta_1 + a_2 \cdot \sin(\theta_1+\theta_2))]}{(a_1 a_2 \sin\theta_2)^2}$$

✓