5. Engineering Economy

Engineering designs are intended to produce good results. In general, the good results are accompanied by undesirable effects including the costs of manufacturing or construction. Selecting the best design from a set of technologically feasible alternatives, or deciding whether or not to implement a proposed design, requires the engineer to anticipate and compare the good and bad outcomes. If outcomes are evaluated in dollars and if "good" is defined as positive monetary value, then design decisions may be guided by the techniques known as engineering economy. Decisions based solely on engineering economy may be guaranteed to result in maximum goodness only if all outcomes are anticipated and can be monetized (measured in dollars).

5.1 Value and Interest

"Value" is not synonymous with "amount." The value of an amount of money depends on when the amount is received or spent. For example, the promise that you will be given a dollar one year from now is of less value to you than a dollar received today. The difference between the anticipated amount and its current value is called "interest" and is frequently expressed as a time rate. If an interest rate of 10% per year is used, the expectation of receiving $1.00 one year hence has a value now of about $0.91. In engineering economy, interest usually is stated in percent per year. If no time unit is given, "per year" is assumed.

EXAMPLE 5.1

What amount must be paid in two years to settle a current debt of $1,000 if the interest rate is 6%?

Solution. Value after one year $= 1000 + 1000 \times 0.06$
$= 1000\,(1 + 0.06)$
$= \$1060$

Value after two years $= 1060 + 1060 \times 0.06$
$= 1000\,(1 + 0.06)^2$
$= \$1124$

Hence, $1124 must be paid in two years to settle the debt.

5.2 Cash Flow Diagrams

As an aid to analysis and communication, an engineering economy problem may be represented graphically by a horizontal time axis and vertical vectors representing dollar amounts. The cash flow diagram for Ex. 5.1 is sketched in Fig. 5.1 on the next page. Income is up and expenditures are down. It is important to pick a point of view and stick with it. For example, the vectors in Fig. 5.1 would have been reversed if the point of view of the lender had been adopted.

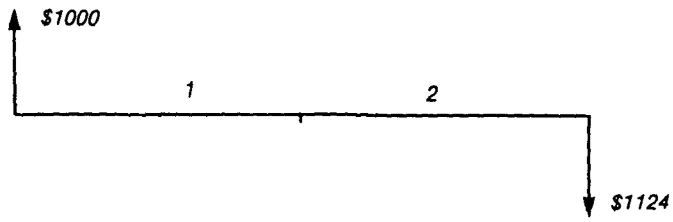


Figure 5.1. Cash flow diagram for Example 5.1.

It is a good idea to draw a cash flow diagram for every engineering economy problem that involves amounts occurring at different times.

In engineering economy, amounts are almost always assumed to occur at the ends of years. Consider, for example, the value today of the future operating expenses of a truck. The costs probably will be paid in varied amounts scattered throughout each year of operation, but for computational ease the expenses in each year are represented by their sum (computed without consideration of interest) occurring at the end of the year. The error introduced by neglecting interest for partial years usually is insignificant compared to uncertainties in the estimates of future amounts.

5.3 Cash Flow Patterns

Engineering economy problems involve the following four patterns of cash flow both separately and in combination.

P-pattern: A single amount P occurring at the beginning of n years. P frequently represents "present" amounts.

F-pattern: A single amount F occurring at the end of n years. F frequently represents "future" amounts.

A-pattern: Equal amounts A occurring at the ends of n years. The A-pattern frequently is used to represent "annual" amounts.

G-pattern: End-of-year amounts increasing by an equal annual gradient G. Note that the first amount occurs at the end of the second year. G is the abbreviation of "gradient."

The four cash flow patterns are illustrated in Fig. 5.2.

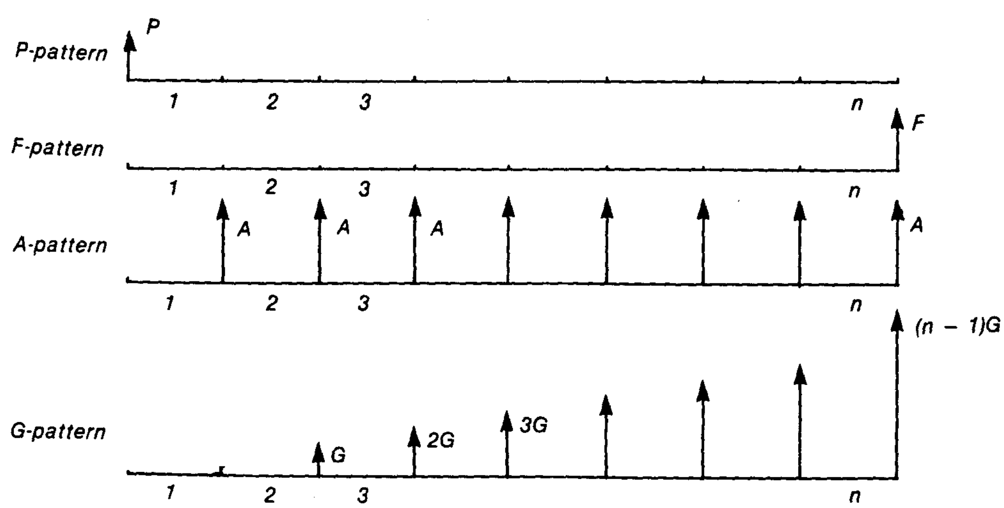


Figure 5.2. Four cash flow patterns.

5.4 Equivalence of Cash Flow Patterns

Two cash flow patterns are said to be equivalent if they have the same value. Most of the computational effort in engineering economy problems is directed at finding a cash flow pattern that is equivalent to a combination of other patterns. Ex. 5.1 can be thought of as finding the amount in an F-pattern that is equivalent to $1,000 in a P-pattern. The two amounts are proportional, and the factor of proportionality is a function of interest rate i and number of periods n. There is a different factor of proportionality for each possible pair of the cash flow patterns defined in Section 5.3. To minimize the possibility of selecting the wrong factor, mnemonic symbols are assigned to the factors. For Ex. 5.1, the proportionality factor is written $(F/P)^i_n$ and solution is achieved by evaluating

$$F = (F/P)^i_n\, P$$

To analysts familiar with the cancelling operation of algebra, it is apparent that the correct factor has been chosen. However, the letters in the parentheses together with the sub- and super-scripts constitute a single symbol; therefore, the cancelling operation is not actually performed. Table 5.1 lists symbols and formulas for commonly used factors. Table 5.2, located at the end of this chapter, presents a convenient way to find numerical values of interest factors. Those values are tabulated for selected interest rates i and number of interest periods n; linear interpolation for intermediate values of i and n is acceptable for most situations.

TABLE 5.1 Formulas for Interest Factors

Symbol	To Find	Given	Formula
$(F/P)^i_n$	F	P	$(1+i)^n$
$(P/F)^i_n$	P	F	$\frac{1}{(1+i)^n}$
$(A/P)^i_n$	A	P	$\frac{i(1+i)^n}{(1+i)^n-1}$
$(P/A)^i_n$	P	A	$\frac{(1+i)^n-1}{i(1+i)^n}$
$(A/F)^i_n$	A	F	$\frac{i}{(1+i)^n-1}$
$(F/A)^i_n$	F	A	$\frac{(1+i)^n-1}{i}$
$(A/G)^i_n$	A	G	$\frac{1}{i}-\frac{n}{(1+i)^n-1}$
$(F/G)^i_n$	F	G	$\frac{1}{i}\left[\frac{(1+i)^n-1}{i}-n\right]$
$(P/G)^i_n$	P	G	$\frac{1}{i}\left[\frac{(1+i)^n-1}{i(1+i)^n}-\frac{n}{(1+i)^n}\right]$

EXAMPLE 5.2

Derive the formula for $(F/P)^i_n$.

Solution. For $n = 1$,

$$F = (1 + i)\,P$$

that is,

$$(F/P)^i_1 = (1 + i)^1$$

For any n,

$$F = (1 + i)\,(F/P)^i_{n-1}\,P$$

that is,

$$(F/P)^i_n = (1 + i)\,(F/P)^i_{n-1}$$

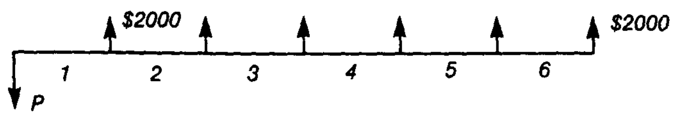
By induction,

$$(F/P)^i_n = (1 + i)^n$$

EXAMPLE 5.3

A new widget twister, with a life of six years, would save $2,000 in production costs each year. Using a 12% interest rate, determine the highest price that could be justified for the machine. Although the savings occur continuously throughout each year, follow the usual practice of lumping all amounts at the ends of years.

Solution. First, sketch the cash flow diagram.



The cash flow diagram indicates that an amount in a P-pattern must be found that is equivalent to $2,000 in an A-pattern. The corresponding equation is

$$P = (P/A)^i_n A$$

$$= (P/A)^{12\%}_6\, 2000$$

Table 5.2 is used to evaluate the interest factor for $i = 12\%$ and $n = 6$:

$$P = 4.1114 \times 2000$$

$$= \$8223$$

EXAMPLE 5.4

How soon does money double if it is invested at 8% interest?

Solution. Obviously, this is stated as

$$F = 2\,P$$

Therefore,

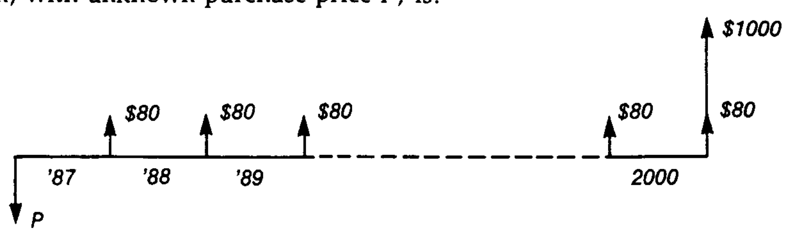
$$(F/P)_n^{8\%} = 2$$

In the 8% interest table, the tabulated value for (F/P) that is closest to 2 corresponds to $n = 9$ years.

EXAMPLE 5.5

Find the value in 1987 of a bond described as "Acme 8% of 2000" if the rate of return set by the market for similar bonds is 10%.

Solution. The bond description means that the Acme Company has an outstanding debt that it will repay in the year 2000. Until then, the company will pay out interest on that debt at the 8% rate. Unless otherwise stated, the principal amount of a single bond is $1000. If it is assumed that the debt is due December 31, 2000, interest is paid every December 31, and the bond is purchased January 1, 1987, then the cash flow diagram, with unknown purchase price P, is:



The corresponding equation is

$$P = (P/A)_{14}^{10\%}\,80 + (P/F)_{14}^{10\%}\,1000$$

$$= 7.3667 \times 80 + 0.2633 \times 1000$$

$$= \$853$$

That is, to earn 10% the investor must buy the 8% bond for $853, a "discount" of $147. Conversely, if the market interest rate is less than the nominal rate of the bond, the buyer will pay a "premium" over $1000.

The solution is approximate because bonds usually pay interest semiannually, and $80 at the end of the year is not equivalent to $40 at the end of each half year. But the error is small and is neglected.

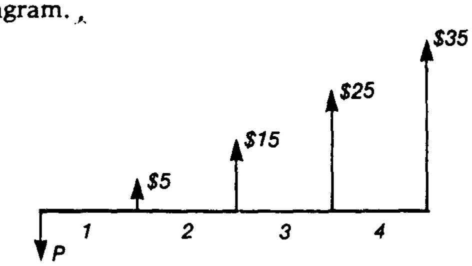
EXAMPLE 5.6

You are buying a new television. From past experience you estimate future repair costs as:

First Year	$ 5
Second Year	15
Third Year	25
Fourth Year	35

The dealer offers to sell you a four-year repair contract for $60. You require at least a 6% interest rate on your investments. Should you invest in the repair contract?

Solution. Sketch the cash flow diagram.



The known cash flows can be represented by superposition of a $5 *A*-pattern and a $10 *G*-pattern. Verify that statement by drawing the two patterns. Now it is clear why the standard *G*-pattern is defined to have the first cash flow at the end of the second year. Next, the equivalent amount *P* is computed:

$$P = (P/A)_4^{6\%} A + (P/G)_4^{6\%} G$$

$$= 3.4651 \times 5 + 4.9455 \times 10$$

$$= \$67$$

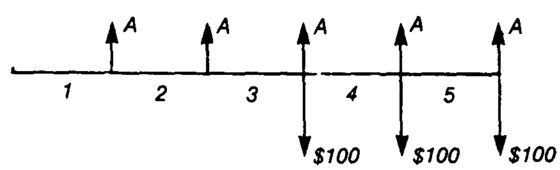
Since the contract can be purchased for less than $67, the investment will earn a rate of return greater than the required 6%. Therefore, you should purchase the contract.

If the required interest rate had been 12%, the decision would be reversed. This demonstrates the effect of required interest rate on decision-making. Increasing the required rate reduces the number of acceptable investments.

EXAMPLE 5.7

Compute the annual equivalent repair costs over a 5-year life if a typewriter is warranted for two years and has estimated repair costs of $100 annually. Use $i = 10\%$.

Solution. The cash flow diagram appears as:



There are several ways to find the 5-year *A*-pattern equivalent to the given cash flow. One of the more efficient methods is to convert the given 3-year *A*-pattern to an *F*-pattern, and then find the 5-year *A*-pattern that is equivalent to that *F*-pattern. That is,

$$A = (A/F)_5^{10\%} (F/A)_3^{10\%} 100$$

$$= \$54$$

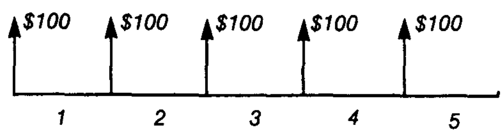
5.5 Unusual Cash Flows and Interest Periods

Occasionally an engineering economy problem will deviate from the year-end cash flow and annual compounding norm. The examples in this section demonstrate how to handle these situations.

EXAMPLE 5.8

PAYMENTS AT BEGINNINGS OF YEARS

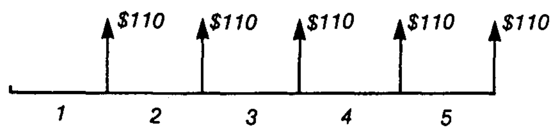
Using 10% interest rate, find the future equivalent of:



Solution. Shift each payment forward one year. That is,

$$A = (F/P)_{1}^{10\%}\ 100 = \$110$$

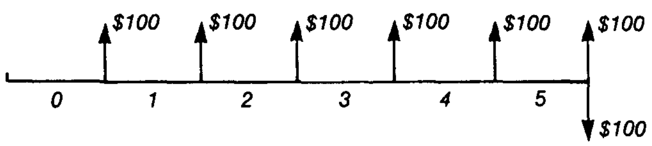
This converts the series to the equivalent A-pattern:



and the future equivalent is found to be

$$F = (F/A)_{5}^{10\%}\ 110 = \$672$$

Alternative Solution. Convert to a six-year series:



The future equivalent is

$$F = (F/A)_{6}^{10\%}\ 100 - 100 = \$672$$

EXAMPLE 5.9

SEVERAL INTEREST AND PAYMENT PERIODS PER YEAR

Compute the present value of eighteen monthly payments of $100 each, where interest is 1% per month.

Solution. The present value is computed as

$$P = (P/A)_{18}^{1\%}\ 100 = \$1640$$

EXAMPLE 5.10

ANNUAL PAYMENTS BUT INTEREST COMPOUNDED m TIMES PER YEAR

Compute the effective annual interest rate equivalent to 5% nominal annual interest compounded daily. There are 250 banking days in a year.

Solution. The legal definition of nominal annual interest is

$$i_n = m\, i$$

where i is the interest rate per compounding period. For the example,

$$i = i_n/m$$

$$= 0.05/250 = 0.0002 \text{ or } 0.02\% \text{ per day}$$

Because of compounding, the effective annual rate is greater than the nominal rate. By equating (F/P)-factors for one year and m periods, the effective annual rate i_e may be computed as follows:

$$(1 + i_e)^1 = (1 + i)^m$$

$$i_e = (1 + i)^m - 1$$

$$= (1.0002)^{250} - 1 = 0.051266 \text{ or } 5.1266\%$$

EXAMPLE 5.11

CONTINUOUS COMPOUNDING

Compute the effective annual interest rate i_e equivalent to 5% nominal annual interest compounded continuously.

Solution. As m approaches infinity, the value for i_e is found as follows:

$$i_e = e^{mi} - 1$$

$$= e^{0.05} - 1$$

$$= 0.051271 \text{ or } 5.1271\%$$

EXAMPLE 5.12

ANNUAL COMPOUNDING BUT m PAYMENTS PER YEAR

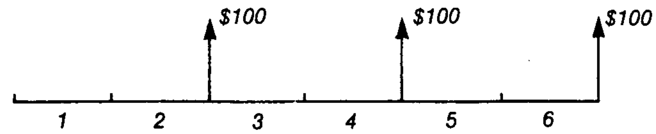
Compute the year-end amount equivalent to twelve end-of-month payments of $10 each. Annual interest rate is 6%.

Solution. The usual simplification in engineering economy is to assume that all payments occur at the end of the year, giving an answer of $120. This approximation may not be acceptable for a precise analysis of a financial agreement. In such cases, the agreement's policy on interest for partial periods must be investigated.

EXAMPLE 5.13

ANNUAL COMPOUNDING BUT PAYMENT EVERY m YEARS

With interest at 10% compute the present equivalent of



Solution. First convert each payment to an A-pattern for the m preceding years. That is,

$$A = (A/F)_{2}^{10\%}\ 100$$

$$= \$47.62$$

Then, convert the A-pattern to a P-pattern:

$$P = (P/A)_{6}^{10\%}\ 47.62$$

$$= \$207$$

5.6 Evaluating Alternatives

The techniques of engineering economy assume the objective of maximizing net value. For a business, "value" means after-tax cash flow. For a not-for-profit organization, such as a government agency, value may include non-cash benefits, such as, clean air, improved public health, recreation, to which dollar amounts have been assigned.

Sections 5.7 through 5.17 concern strategies for selecting alternatives such that net value is maximized. The logic of these methods will be clear if the following distinctions are made between two different types of interest rates, and between two different types of relationships among alternatives.

TYPES OF INTEREST RATES

Rate of Return (ROR): The estimated interest rate produced by an investment. It may be computed by finding the interest rate such that the estimated income and non-cash benefits (positive value), and the estimated expenditures and non-cash costs (negative value) sum to a net equivalent value of zero.

Minimum Attractive Rate of Return (MARR): The lowest rate of return that the organization will accept. In engineering economy problems, it is usually a given quantity and may be called, somewhat imprecisely, "interest," "interest rate," "cost of money," or "interest on capital."

TYPES OF ALTERNATIVE SETS

Mutually Exclusive Alternatives: Exactly one alternative must be selected.
Examples: "Shall Main Street be paved with concrete or asphalt?" "In which room will we put the piano?" If a set of alternatives is mutually exclusive, it is important to determine whether the set includes the null (do nothing) alternative. Serious consequences can arise from failure to recognize the null alternative.

Independent Alternatives: It is possible (but not necessarily economical) to select any number of the available alternatives.
Examples: "Which streets should be paved this year?" "Which rooms shall we carpet?"

5.7 Annual Equivalent Cost Comparisons

The estimated income and benefits (positive) and expenditures and costs (negative) associated with an alternative are converted to the equivalent *A*-pattern using an interest rate equal to *MARR*. The *A*-value is the annual net equivalent value (*ANEV*) of that alternative. If the alternatives are mutually exclusive, the one with the largest *ANEV* is selected. If the alternatives are independent, all that have positive *ANEV* are selected.

EXAMPLE 5.14

A new cap press is needed. Select the better of the two available models described below. *MARR* is 10%.

Model	Price	Annual Maintenance	Salvage Value	Life
Reliable	11,000	1,000	1,000	10 yrs.
Quicky	4,000	1,500	0	5 yrs.

Solution. The *ANEV* is calculated for each model:

$$\text{Reliable: } ANEV = -(A/P)_{10}^{10\%}\ 11000 - 1000 + (A/F)_{10}^{10\%}\ 1000$$

$$= -\$2730.$$

$$\text{Quicky: } ANEV = -(A/P)_{5}^{10\%}\ 4000 - 1500$$

$$= -\$2560.$$

Negative *ANEV* indicates a rate of return less than *MARR*. However, these alternatives are mutually exclusive and the null is not available. The problem is one of finding the less costly way to perform a necessary function. Therefore, *Quicky* is selected. If *MARR* had been much lower, *Reliable* would have been selected. By setting the *MARR* relatively high, the organization is indicating that funds are not available to invest now in order to achieve savings in the future.

5.8 Present Equivalent Cost Comparisons

The estimated income and benefits (positive), and expenditures and costs (negative) associated with an alternative are converted to the equivalent *P*-pattern using an interest rate equal to *MARR*. The *P*-value is the present net equivalent value (*PNEV*) of that alternative. If the alternatives are mutually exclusive, the one with the largest *PNEV* is selected. If the alternatives are independent, all that have positive *PNEV* are selected.

The present equivalent cost method requires that alternatives be evaluated over the same span of time. If their lives are not equal, the lowest common multiple of the lives is used for the time span, with each alternative repeated to fill the span. A variation, called the "capitalized cost method," computes the *PNEV* for repeated replacement of the alternatives for an infinite time span.

PNEV is also called "life cycle cost," "present worth," "capital cost," and "venture worth."

EXAMPLE 5.15

Repeat Ex. 5.14 using the present equivalent cost method.

Solution. The *PNEV* is calculated for each model

$$\textit{Reliable:}\ PNEV = -\ 11000 - (P/A)_{10}^{10\%}\ 1000 + (P/F)_{10}^{10\%}\ 1000$$

$$= -\ \$16{,}800$$

$$\textit{Quicky:}\ PNEV = -\ 4000 - (P/F)_{5}^{10\%}\ 4000 - (P/A)_{10}^{10\%}\ 1500$$

$$= -\ \$15{,}700$$

Note that *Quicky* was replaced in order to fill the ten-year time span. As in Ex. 5.14, *Quicky* is selected. The two methods always will give the same decision if used correctly. Observe that for both alternatives $PNEV = (P/A)_{10}^{10\%}\ ANEV$.

5.9 Incremental Approach

For a set of mutually exclusive alternatives, only the differences in amounts need to be considered. Compute either the *ANEV* or the *PNEV* and base the decision on the sign of that value.

EXAMPLE 5.16

Repeat Ex. 5.14 using an incremental present net equivalent value approach.

Solution. *Reliable* costs $7000 more than *Quicky* but saves $500 each year in maintenance expenses and eliminates the need for a $4000 replacement after five years. In addition, *Reliable* has a $1000 salvage value whereas *Quicky* has none.

Reliable − *Quicky:*

$$PNEV = -\ 7000 + (P/A)_{10}^{10\%}\ 500 + (P/F)_{5}^{10\%}\ 4000 + (P/F)_{10}^{10\%}\ 1000$$

$$= -\ \$1060$$

The negative result dictates selection of *Quicky*. That is, the additional initial cost required to purchase *Reliable* is not justified.

5.10 Rate of Return Comparisons

The expression for *ANEV* or *PNEV* is formulated and then solved for the interest rate that will give a zero *ANEV* or *PNEV*. This interest rate is the rate of return (*ROR*) of the alternative. To apply the rate of return method to mutually exclusive alternatives requires incremental comparison of each possible pair of alternatives; increments of investment are accepted if their rates of return exceed *MARR*. For independent alternatives, all those with *ROR* exceeding *MARR* are accepted. The rate of return method permits conclusions to be stated as functions of *MARR*, which is useful if *MARR* has not been determined precisely.

EXAMPLE 5.17

A magazine subscription costs $5.00 for one year or $8.00 for two years. If you want to receive the magazine for at least two years, which alternative is better?

Solution. The two-year subscription requires an additional initial investment of $3.00 and eliminates the payment of $5.00 one year later. The rate of return formulation is

$$PNEV = 0$$

$$-3 + 5\,(P/F)_1^{\,i} = 0$$

The solution for i is as follows:

$$-3 + 5\,\frac{1}{(1+i)} = 0$$

$$i = 0.67 \text{ or } 67\%$$

Therefore, if your *MARR* is less than 67%, subscribe for two years.

EXAMPLE 5.18

Repeat Ex. 5.14 using the rate of return method.

Solution. Use the incremental expression derived in Ex. 5.16, but set *PNEV* equal to zero and use interest rate as the unknown.

$$-\,7000 + (P/A)_{10}^{i}\,500 + (P/F)_{5}^{i}\,4000 + (P/F)_{10}^{i}\,1000 = 0$$

By trial and error, the interest rate is found to be 6.6%. Therefore, *Reliable* is preferred if, and only if, *MARR* is less than 6.6%.

5.11 Benefit/Cost Comparisons

The benefit/cost ratio is determined from the formula:

$$\frac{B}{C} = \frac{\text{Uniform net annual benefits}}{\text{Annual equivalent of initial cost}}$$

where *MARR* is used in computing the *A*-value in the denominator. As with the rate of return method, mutually exclusive alternatives must be compared incrementally, the incremental investment being accepted if the benefit/cost ratio exceeds unity. For independent alternatives, all those with benefit/cost ratios exceeding unity are accepted.

Note that the only pertinent fact about a benefit/cost ratio is whether it exceeds unity. This is illustrated by the observation that a project with a ratio of 1.1 may provide greater net benefit than a project with a ratio of 10 if the investment in the former project is much larger than the investment in the latter. It is incorrect to rank mutually exclusive alternatives by their benefit/cost ratios as determined by comparing each alternative to the null (do nothing) alternative.

The benefit/cost ratio method will give the same decisions as the rate of return method, present equivalent cost method and annual equivalent cost method if the following conditions are met:

1. Each alternative is comprised of an initial cost and uniform annual benefit.

2. The form of the benefit/cost ratio given above is used without deviation.

EXAMPLE 5.19

A road resurfacing project costs $200,000, lasts five years and saves $100,000 annually in patching costs. *MARR* is 10%. Should the road be resurfaced?

Solution. The benefit/cost ratio is

$$\frac{B}{C} = \frac{100{,}000}{(A/P)_{5}^{10\%}\ 200{,}000} = 1.9$$

Since the ratio exceeds unity, the resurfacing is justified.

5.12 A Note on *MARR*

In engineering economy examination problems, *MARR* is a given quantity. However, the following discussion of the determination of *MARR* will help clarify the logic underlying the various comparison methods.

In general, an organization will be able to identify numerous opportunities to spend money now that will result in future returns. For each of these independent investment opportunities, an expected rate of return can be estimated. Similarly, the organization will be able to find numerous sources of funds for investment. Associated with each source of funds is an interest rate. If the source is a loan, the associated interest rate is simply that charged by the lender. Funds generated by operations of the organization, or provided by its owners (if the organization is a business), or extracted from taxpayers (if the organization is a government agency) can be thought of as being borrowed from the owners or taxpayers. Therefore, such funds can be assigned a fictitious interest rate, which should not be less than the maximum rate of return provided by other opportunities in which the owners or taxpayers might invest.

Value will be maximized if the rates of return of all the selected investments exceed the highest interest rate charged for the money borrowed, and if every opportunity has been taken to invest at a rate of return exceeding that for which money can be borrowed. The marginal dollar is invested at a rate of return equal to the interest rate at which it was borrowed. That rate is the Minimum Attractive Rate of Return. No investments should be made that pay rates of return less than *MARR*, and no loans should be taken that charge interest rates exceeding *MARR*. Furthermore, the organization should exploit all opportunities to borrow money at interest rates less than *MARR* and invest it at rates of return exceeding *MARR*.

To estimate *MARR* precisely would require the ability to foresee the future, or at least to predict all future investment and borrowing opportunities and their associated rates. A symptom of *MARR* being set too low is insufficient funds for all the investments that appear to be acceptable. Conversely, if *MARR* has been set too high, some investments will be rejected that would have been profitable.

5.13 Replacement Problems

How frequently should a particular machine be replaced? This type of problem can be approached by varying the life n. For each value of n, the annual costs and salvage value are estimated, and then the *ANEV* is computed. The value of n resulting in the smallest annual equivalent cost is the optimum, or economic, life of the machine. This approach is complicated by technological improvements in replacement machinery, which may make it advantageous to replace a machine before the end of its economic life. In practice, technological advances are difficult to anticipate.

Another form of the replacement problem asks if an existing asset should be replaced by a new (and possibly different) one. Again, the annual equivalent cost method is recommended. The *ANEV* of the replacement is computed, using its economic life for n. However, the annual cost of the existing asset is simply the estimated expense of one more year of operation. This strategy is based on the assumption that annual costs of the existing asset increase monotonically as it ages.

5.14 Always Ignore the Past

Engineering economy, and decision-making in general, deals with alternatives. But there is only one past and it affects all future alternatives equally. Therefore, past costs and income associated with an existing asset should not be included in computations that address the question of replacing the asset. Only the estimated cash flows of the future are relevant.

The mistake of counting past costs is common in everyday affairs. For example, a student may say, "I paid $40 for this textbook so I will not sell it for $20." A more rational approach would be to compare the highest offered price to the value of retaining the text.

EXAMPLE 5.20

Yesterday a machine was bought for $10,000. Estimated life is ten years, with no salvage value at that time. Current book value is $10,000. Today a vastly improved model was announced. It costs $15,000, has a ten-year life and no salvage value at that time, but reduces operating costs by $4,000 annually. The current resale value of the older machine has dropped to $1,000 due to this stunning technological advance. Should the old model be replaced with the new model at this time?

Solution. The purchase price of the old machine, its book value, and the loss on the sale of the old machine are irrelevant to the analysis. The incremental cost of the new machine is $14,000 and the incremental income is $4,000 annually. A rate of return comparison is formulated as follows:

$$-14{,}000 + (P/A)^{i}_{10}4000 = 0$$

Solving for rate of return gives $i = 26\%$, indicating that the older machine should be replaced immediately if *MARR* is less than 26%.

5.15 Break-Even Analysis

A break-even point is the value of an independent variable such that two alternatives are equally attractive. For values of the independent variable above the break-even point, one of the alternatives is preferred; for values of the independent variable below the break-even point, the other alternative is preferred. Break-even analysis is particularly useful for dealing with an independent variable that is subject to change or uncertainty since the conclusion of the analysis can be stated as a function of that variable. The rate of return method, as applied to mutually exclusive alternatives, is an example of break-even analysis. The independent variable is *MARR*.

EXAMPLE 5.21

An item can be manufactured by hand for $5. Alternatively, the item can be produced by a machine at a fixed annual equivalent cost of $4,000 plus a variable cost of $1 per item. Assume that the cost of laying off and hiring workers is zero. For each of the two manufacturing processes, answer the following questions:

a) For what production rate is one method more economical than the other?

b) If the item is sold for $6, how many must be sold to make a profit?

c) How low must the price fall, in the short term, before production is discontinued?

Solution.

a) Let P be production rate in units per year. Production costs for the two processes are equated:

$$\text{Cost by machine} = \text{Cost manually}$$

$$4000 + 1\,P = 5\,P$$

$$\therefore P = 1000$$

If annual production is expected to be less than 1000 units, the manual process is more economical. For production rates exceeding 1000 units per year, the machine process is preferred.

b) Setting profit equal to zero is expressed as

$$\text{gross income} - \text{cost} = 0$$

$$\text{Manual production: } 6\,P - 5\,P = 0$$

$$\therefore P = 0$$

$$\text{Machine production: } 6\,P - (4000 + 1\,P) = 0$$

$$\therefore P = 800$$

With price maintained at $6, the mechanized operation will be unprofitable if production rate is less than 800 units per year, but the manual operation is profitable at all production rates.

c) Manual production becomes unprofitable if the price drops below $5, and production will cease at that level. For the machine, the $4,000 cost continues whether or not the machine is running. Incremental income is generated so long as the price stays above the variable (per item) cost. Therefore, production will continue at any price over $1, even though a net loss may be sustained. Of course, if it appears that the price and production rate will not soon increase sufficiently to provide a profit, then the operation will be terminated.

5.16 Income Tax and Depreciation

Businesses pay to the federal government a tax that is a proportion of taxable income. Taxable income is gross revenue less operating costs (wages, cost of materials, etc.), interest payments on debts, and depreciation. Depreciation is different from the other deductions in that it is not a cash flow.

Depreciation is an accounting technique for charging the initial cost of an asset against two or more years of production. For example, if you buy a $15,000 truck for use in your construction business, deducting its total cost from income during the year of purchase gives an unrealistically low picture of income for that year, and an unrealistically high estimate of income for the succeeding years during which you use the truck. A more level income history would result if you deducted $5,000 per year for three years. In fact, the Internal Revenue Service (IRS) requires that most capital assets used in business be depreciated over a number of years rather than being deducted as expenses during the year of purchase.

An asset is depreciable if it is used to produce income, has a determinable life greater than one year, and decays, wears out, becomes obsolete, or gets used up. Examples are: tools, production machinery, computers, office equipment, buildings, patents, contracts, franchises, and livestock raised for wool, eggs, milk or breeding. Non-depreciable assets include personal residence, land, natural resources, annual crops, livestock raised for sale or slaughter, and items intended primarily for resale such as stored grain and the merchandise in a department store.

Since depreciation is not a cash flow, it will not directly enter an engineering economy analysis. However, depreciation must be considered when estimating future income taxes, which are cash flows.

For assets placed in service after 1980, the IRS requires that depreciation be computed by the accelerated cost recovery system (*ACRS*). But older methods may still show up in engineering economy problems. Therefore, the three methods permitted before 1981 are helpful to know. The following notation will be used in defining methods for computing depreciation:

B — The installed first cost, or basis.

n — Recovery period in years.

D_x — Depreciation in year x.

V_x — Undepreciated balance at the end of year x, also called book value. $V_0 = B$.

V_n — Estimated salvage at age n.

In computing depreciation there is no attempt to equate book value with resale value, productive worth, or any other real figure. A business is not obliged to keep an asset for exactly n years, nor to sell it for exactly its book value or estimated salvage value. These, then, are the four depreciation methods:

1. *Accelerated Cost Recovery System (ACRS):* An asset is classed as having a recovery period n of 3, 5, 10 or 15 years using IRS guidelines. For each class, a set of annual rates R_x is specified by the IRS. With 3-year property, for example, $R_1 = 0.25$, $R_2 = 0.38$, $R_3 = 0.37$. Depreciation is calculated by

$$D_x = R_x B.$$

   By definition, the salvage value in the *ACRS* is zero.

2. *Straight Line Depreciation:* Depreciation is the same for every year and is calculated as

$$D_x = (B - V_n)/n$$

   In this and the next two methods, n and V_n are estimated by the taxpayer at time of purchase and are required to be realistic.

3. *Sum of Years' Digits:* This method, the one that follows, and most instances of the *ACRS* method are said to be "accelerated" because they reduce book value more rapidly than does the straight line method. In general, accelerated depreciation is desirable because it produces larger tax deductions (and, therefore, larger after-tax cash flows) in the early years.

   The sum of the years' digits method uses the relationship

$$D_x = (B - V_n)\frac{\text{(years of life remaining at start of year } x\text{)}}{\text{(sum of digits of all years of life)}}$$

$$= (B - V_n)(n - x + 1)/(0.5\,n^2 + 0.5\,n)$$

4. *Declining Balance:* Depreciation is taken as a proportion of book value:

$$D_x = V_{x-1}\,C/n$$

   For values of C equaling 1.25, 1.5 and 2 the method is called, respectively: 125% declining balance, 150% declining balance, and double declining balance. The formula may result in book values less than estimated salvage value, but this is not permitted by the IRS.

EXAMPLE 5.22

The purchase price of a light truck is $15,000, its recovery period is three years, and it can be sold for an estimated $1,500 at that time. Compute the depreciation schedules using each of the methods described.

Solution.

Accelerated Cost Recovery System

Year	Depreciation	Book Value
		$15,000
1	0.25 × 15,000 = $3750	$11,250
2	0.38 × 15,000 = $5700	$ 5,550
3	0.37 × 15,000 = $5550	0

Straight Line

Year	Depreciation	Book Value
		$15,000
1	(15,000 − 1500)/3 = $4500	$10,500
2	$4500	$ 6,000
3	$4500	$ 1,500

Sum of Years' Digit

Year	Depreciation	Book Value
		$15,000
1	(15,000 − 1500) 3/6 = $6750	$ 8,250
2	13,500 × 2/6 = $4500	$ 3,750
3	13,500 × 1/6 = $2250	$ 1,500

Double Declining Balance

Year	Depreciation	Book Value
		$15,000
1	15,000 × 2/3 = $10,000	$ 5,000
2	5,000 × 2/3 = $ 3,333	$ 1,667
3	$167	$ 1,500

In the third year, the formula would have resulted in a book value less than the estimated salvage value, so the formula was abandoned.

5.17 Inflation

The "buying power" of money changes with time. A decline in "buying power" is experienced as a general increase in prices and is called "inflation."

Inflation, if it is anticipated, can be exploited by fixing costs and allowing income to increase. A manufacturing business can fix its costs by entering long-term contracts for materials and wages, by purchasing materials long before they are needed, and by stockpiling its product for sale later. Income is allowed to respond to inflation by avoiding long-term contracts for the product. Borrowing becomes more attractive if inflation is expected since the debt will be paid with the less valuable cash of the future.

EXAMPLE 5.23

A machine having a five-year life can replace a worker now earning $10,000 per year who is subject to 5% annual "cost of living" increases. Operating and maintenance costs for the machine are negligible. *MARR* is 10%. Find the maximum price that can be justified for the machine if:

a) general price inflation is 5%, and

b) general price inflation is zero.

Solution.

a) Although the worker gets a larger amount of money each year, his raises are exactly matched by increased prices, including those of his employer's product. "Buying power" of his annual wage remains equal to the current value of $10,000. Hence, the maximum justifiable price for the machine is

$$P = (P/A)^{10\%}_{5}\ 10{,}000 = \$37{,}908$$

b) The maximum justifiable price of the machine is equal to the present equivalent value of the annual amounts of the wage:

$$(P/F)^{10\%}_{1}\ (1.05)\ 10{,}000 = \$\ 9{,}545$$

$$(P/F)^{10\%}_{2}\ (1.05)^2\ 10{,}000 = \$\ 9{,}112$$

$$(P/F)^{10\%}_{3}\ (1.05)^3\ 10{,}000 = \$\ 8{,}697$$

$$(P/F)^{10\%}_{4}\ (1.05)^4\ 10{,}000 = \$\ 8{,}302$$

$$(P/F)^{10\%}_{5}\ (1.05)^5\ 10{,}000 = \underline{\$\ 7{,}925}$$

$$\therefore P = \$43{,}581$$

EXAMPLE 5.24

Recompute the value, in terms of 1987 "buying power," of the "Acme 8% of 2000" bond discussed in Ex. 5.5, but assume 6% annual inflation.

Solution. The cash flow for each year must be divided by an inflation factor as well as multiplied by an interest factor, and then the factored cash flows are added:

$$(P/F)^{10\%}_{1} \quad 80/(1.06) = \$\ 69$$

$$(P/F)^{10\%}_{2} \quad 80/(1.06)^2 = \$\ 59$$

$$(P/F)^{10\%}_{3} \quad 80/(1.06)^3 = \$\ 50$$

$$\vdots$$

$$(P/F)^{10\%}_{13} \quad 80/(1.06)^{13} = \$\ 11$$

$$(P/F)^{10\%}_{14} \quad 80/(1.06)^{14} = \$\ \ 9$$

$$(P/F)^{10\%}_{14} \quad 1000/(1.06)^{14} = \underline{\$116}$$

$$P = \$541$$

Note that investors can account for anticipated inflation simply by using increased values of *MARR*. A *MARR* of 16.6% gives the same conclusions as a *MARR* of 10% with 6% inflation.

TABLE 5.2. Compound Interest Factors

$i = 1/2\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9950	0.995	0.000	1.005	1.000	1.0050	1.0000	0.000	1
2	.9901	1.895	0.990	1.010	2.005	0.5038	0.4988	0.499	2
3	.9851	2.970	2.960	1.015	3.015	0.3367	0.3317	0.997	3
4	.9802	3.950	5.901	1.020	4.030	0.2531	0.2481	1.494	4
5	.9754	4.926	9.803	1.025	5.050	0.2030	0.1980	1.990	5
6	.9705	5.896	14.655	1.030	6.076	0.1696	0.1646	2.485	6
7	.9657	6.862	20.449	1.036	7.106	0.1457	0.1407	2.980	7
8	.9609	7.823	27.176	1.041	8.141	0.1278	0.1228	3.474	8
9	.9561	8.779	34.824	1.046	9.182	0.1139	0.1089	3.967	9
10	.9513	9.730	43.386	1.051	10.228	0.1028	0.0978	4.459	10
11	.9466	10.677	52.853	1.056	11.279	0.0937	0.0887	4.950	11
12	.9419	11.619	63.214	1.062	12.336	0.0861	0.0811	5.441	12
13	.9372	12.556	74.460	1.067	13.397	0.0796	0.0746	5.930	13
14	.9326	13.489	86.583	1.072	14.464	0.0741	0.0691	6.419	14
15	.9279	14.417	99.574	1.078	15.537	0.0694	0.0644	6.907	15
16	.9233	15.340	113.424	1.083	16.614	0.0652	0.0602	7.394	16
17	.9187	16.259	128.123	1.088	17.697	0.0615	0.0565	7.880	17
18	.9141	17.173	143.663	1.094	18.786	0.0582	0.0532	8.366	18
19	.9096	18.082	160.036	1.099	19.880	0.0553	0.0503	8.850	19
20	.9051	18.987	177.232	1.105	20.979	0.0527	0.0477	9.334	20
21	.9006	19.888	195.243	1.110	22.084	0.0503	0.0453	9.817	21
22	.8961	20.784	214.061	1.116	23.194	0.0481	0.0431	10.299	22
23	.8916	21.676	233.677	1.122	24.310	0.0461	0.0411	10.781	23
24	.8872	22.563	254.082	1.127	25.432	0.0443	0.0393	11.261	24
25	.8828	23.446	275.269	1.133	26.559	0.0427	0.0377	11.741	25
26	.8784	24.342	297.228	1.138	27.692	0.0411	0.0361	12.220	26
28	.8697	26.068	343.433	1.150	29.975	0.0384	0.0334	13.175	28
30	.8610	27.794	392.632	1.161	32.280	0.0360	0.0310	14.126	30
∞	0	200.000	40000.0	∞	∞	.0050	0	200.000	∞

$i = 3/4\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	0.99256	0.9925	0.0000	1.0075	1.0000	1.00750	1.00000	0.0000	1
2	0.98517	1.9777	0.9851	1.0150	2.0075	0.50563	0.49813	0.4981	2
3	0.97783	2.9555	2.9408	1.0226	3.0225	0.33835	0.33085	0.9950	3
4	0.97055	3.9261	5.8525	1.0303	4.0452	0.25471	0.24721	1.4906	4
5	0.96333	4.8894	9.7058	1.0380	5.0755	0.20452	0.19702	1.9850	5
6	0.95616	5.8456	14.486	1.0458	6.1136	0.17107	0.16357	2.4782	6
7	0.94904	6.7946	20.180	1.0537	7.1594	0.14717	0.13967	2.9701	7
8	0.94198	7.7366	26.774	1.0616	8.2131	0.12926	0.12176	3.4607	8
9	0.93496	8.6715	34.254	1.0695	9.2747	0.11532	0.10782	3.9501	9
10	0.92800	9.5995	42.606	1.0775	10.344	0.10417	0.09667	4.4383	10
11	0.92109	10.520	51.817	1.0856	11.421	0.09505	0.08755	4.9252	11
12	0.91424	11.434	61.874	1.0938	12.507	0.08745	0.07995	5.4109	12
13	0.90743	12.342	72.763	1.1020	13.601	0.08102	0.07352	5.8954	13
14	0.90068	13.243	84.472	1.1102	14.703	0.07551	0.06801	6.3786	14
15	0.89397	14.137	96.987	1.1186	15.813	0.07074	0.06324	6.8605	15
16	0.88732	15.024	110.29	1.1269	16.932	0.06656	0.05906	7.3412	16
17	0.88071	15.905	124.38	1.1354	18.059	0.06287	0.05537	7.8207	17
18	0.87416	16.779	139.24	1.1439	19.194	0.05960	0.05210	8.2989	18
19	0.86765	17.646	154.86	1.1525	20.338	0.05667	0.04917	8.7759	19
20	0.86119	18.508	171.23	1.1611	21.491	0.05403	0.04653	9.2516	20
21	0.85478	19.362	188.32	1.1698	22.652	0.05165	0.04415	9.7261	21
22	0.84842	20.211	206.14	1.1786	23.822	0.04948	0.04198	10.199	22
23	0.84210	21.053	224.66	1.1875	25.001	0.04750	0.04000	10.671	23
24	0.83583	21.889	243.89	1.1964	26.188	0.04568	0.03818	11.142	24
25	0.82961	22.718	263.80	1.2053	27.384	0.04402	0.03652	11.611	25
26	0.82343	23.542	284.38	1.2144	28.590	0.04248	0.03498	12.080	26
28	0.81122	25.170	327.54	1.2327	31.028	0.03973	0.03223	13.012	28
30	0.79919	26.775	373.26	1.2512	33.502	0.03735	0.02985	13.940	30
∞	0	133.333	17777.8	∞	∞	.0075	0	133.333	∞

TABLE 5.2. Compound Interest Factors (continued)

$i = 1.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9901	0.9901	-0.0000	1.0100	1.0000	1.0100	1.0000	-0.0000	1
2	.9803	1.9704	0.9803	1.0201	2.0100	0.5075	0.4975	0.4975	2
3	.9706	2.9410	2.9215	1.0303	3.0301	0.3400	0.3300	0.9934	3
4	.9610	3.9020	5.8044	1.0406	4.0604	0.2563	0.2463	1.4876	4
5	.9515	4.8534	9.6103	1.0510	5.1010	0.2060	0.1960	1.9801	5
6	.9420	5.7955	14.3205	1.0615	6.1520	0.1725	0.1625	2.4710	6
7	.9327	6.7282	19.9168	1.0721	7.2135	0.1486	0.1386	2.9602	7
8	.9235	7.6517	26.3812	1.0829	8.2857	0.1307	0.1207	3.4478	8
9	.9143	8.5660	33.6959	1.0937	9.3685	0.1167	0.1067	3.9337	9
10	.9053	9.4713	41.8435	1.1046	10.4622	0.1056	0.0956	4.4179	10
11	.8963	10.3676	50.8067	1.1157	11.5668	0.0965	0.0865	4.9005	11
12	.8874	11.2551	60.5687	1.1268	12.6825	0.0888	0.0788	5.3815	12
13	.8787	12.1337	71.1126	1.1381	13.8093	0.0824	0.0724	5.8607	13
14	.8700	13.0037	82.4221	1.1495	14.9474	0.0769	0.0669	6.3384	14
15	.8613	13.8651	94.4810	1.1610	16.0969	0.0721	0.0621	6.8143	15
16	.8528	14.7179	107.2734	1.1726	17.2579	0.0679	0.0579	7.2886	16
17	.8444	15.5623	120.7834	1.1843	18.4304	0.0643	0.0543	7.7613	17
18	.8360	16.3983	134.9957	1.1961	19.6147	0.0610	0.0510	8.2323	18
19	.8277	17.2260	149.8950	1.2081	20.8109	0.0581	0.0481	8.7017	19
20	.8195	18.0456	165.4664	1.2202	22.0190	0.0554	0.0454	9.1694	20
21	.8114	18.8570	181.6950	1.2324	23.2392	0.0530	0.0430	9.6354	21
22	.8034	19.6604	198.5663	1.2447	24.4716	0.0509	0.0409	10.0998	22
23	.7954	20.4558	216.0660	1.2572	25.7163	0.0489	0.0389	10.5626	23
24	.7876	21.2434	234.1800	1.2697	26.9735	0.0471	0.0371	11.0237	24
25	.7798	22.0232	252.8945	1.2824	28.2432	0.0454	0.0354	11.4831	25
26	.7720	22.7952	272.1957	1.2953	29.5256	0.0439	0.0339	11.9409	26
28	.7568	24.3164	312.5047	1.3213	32.1291	0.0411	0.0311	12.8516	28
30	.7419	25.8077	355.0021	1.3478	34.7849	0.0387	0.0287	13.7557	30
∞	.0000	100.000	10 000.0	∞	∞	0.0100	0.0000	100.0000	∞

$i = 2.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9804	0.9804	-0.0000	1.0200	1.0000	1.0200	1.0000	-0.0000	1
2	.9612	1.9416	0.9612	1.0404	2.0200	0.5150	0.4950	0.4950	2
3	.9423	2.8839	2.8458	1.0612	3.0604	0.3468	0.3268	0.9868	3
4	.9238	3.8077	5.6173	1.0824	4.1216	0.2626	0.2426	1.4752	4
5	.9057	4.7135	9.2403	1.1041	5.2040	0.2122	0.1922	1.9604	5
6	.8880	5.6014	13.6801	1.1262	6.3081	0.1785	0.1585	2.4423	6
7	.8706	6.4720	18.9035	1.1487	7.4343	0.1545	0.1345	2.9208	7
8	.8535	7.3255	24.8779	1.1717	8.5830	0.1365	0.1165	3.3961	8
9	.8368	8.1622	31.5720	1.1951	9.7546	0.1225	0.1025	3.8681	9
10	.8203	8.9826	38.9551	1.2190	10.9497	0.1113	0.0913	4.3367	10
11	.8043	9.7868	46.9977	1.2434	12.1687	0.1022	0.0822	4.8021	11
12	.7885	10.5753	55.6712	1.2682	13.4121	0.0946	0.0746	5.2642	12
13	.7730	11.3484	64.9475	1.2936	14.6803	0.0881	0.0681	5.7231	13
14	.7579	12.1062	74.7999	1.3195	15.9739	0.0826	0.0626	6.1786	14
15	.7430	12.8493	85.2021	1.3459	17.2934	0.0778	0.0578	6.6309	15
16	.7284	13.5777	96.1288	1.3728	18.6393	0.0737	0.0537	7.0799	16
17	.7142	14.2919	107.5554	1.4002	20.0121	0.0700	0.0500	7.5256	17
18	.7002	14.9920	119.4581	1.4282	21.4123	0.0667	0.0467	7.9681	18
19	.6864	15.6785	131.8139	1.4568	22.8406	0.0638	0.0438	8.4073	19
20	.6730	16.3514	144.6003	1.4859	24.2974	0.0612	0.0412	8.8433	20
21	.6598	17.0112	157.7959	1.5157	25.7833	0.0588	0.0388	9.2760	21
22	.6468	17.6580	171.3795	1.5460	27.2990	0.0566	0.0366	9.7055	22
23	.6342	18.2922	185.3309	1.5769	28.8450	0.0547	0.0347	10.1317	23
24	.6217	18.9139	199.6305	1.6084	30.4219	0.0529	0.0329	10.5547	24
25	.6095	19.5235	214.2592	1.6406	32.0303	0.0512	0.0312	10.9745	25
26	.5976	20.1210	229.1987	1.6734	33.6709	0.0497	0.0297	11.3910	26
28	.5744	21.2813	259.9392	1.7410	37.0512	0.0470	0.0270	12.2145	28
30	.5521	22.3965	291.7164	1.8114	40.5681	0.0446	0.0246	13.0251	30
∞	.0000	50.0000	2500.0000	∞	∞	0.0200	0.0000	50.0000	∞

TABLE 5.2. Compound Interest Factors (continued)

$i = 3\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	0.97087	0.9708	0.0000	1.0300	1.0000	1.03000	1.00000	0.0000	1
2	0.94260	1.9134	0.9426	1.0609	2.0300	0.52261	0.49261	0.4926	2
3	0.91514	2.8286	2.7728	1.0927	3.0909	0.35353	0.32353	0.9803	3
4	0.88849	3.7171	5.4383	1.1255	4.1836	0.26903	0.23903	1.4630	4
5	0.86261	4.5797	8.8887	1.1592	5.3091	0.21835	0.18835	1.9409	5
6	0.83748	5.4171	13.076	1.1940	6.4684	0.18460	0.15460	2.4138	6
7	0.81309	6.2302	17.954	1.2298	7.6624	0.16051	0.13051	2.8818	7
8	0.78941	7.0196	23.480	1.2667	8.8923	0.14246	0.11246	3.3449	8
9	0.76642	7.7861	29.611	1.3047	10.159	0.12843	0.09843	3.8031	9
10	0.74409	8.5302	36.308	1.3439	11.463	0.11723	0.08723	4.2565	10
11	0.72242	9.2526	43.533	1.3842	12.807	0.10808	0.07808	4.7049	11
12	0.70138	9.9540	51.248	1.4257	14.192	0.10046	0.07046	5.1485	12
13	0.68095	10.635	59.419	1.4685	15.617	0.09403	0.06403	5.5872	13
14	0.66112	11.296	68.014	1.5125	17.086	0.08853	0.05853	6.0210	14
15	0.64186	11.937	77.000	1.5579	18.598	0.08377	0.05377	6.4500	15
16	0.62317	12.561	86.347	1.6047	20.156	0.07961	0.04961	6.8742	16
17	0.60502	13.166	96.028	1.6528	21.761	0.07595	0.04595	7.2935	17
18	0.58739	13.753	106.01	1.7024	23.414	0.07271	0.04271	7.7081	18
19	0.57029	14.323	116.27	1.7535	25.116	0.06981	0.03981	8.1178	19
20	0.55368	14.877	126.79	1.8061	26.870	0.06722	0.03722	8.5228	20
21	0.53755	15.415	137.55	1.8602	28.676	0.06487	0.03487	8.9230	21
22	0.52189	15.936	148.50	1.9161	30.536	0.06275	0.03275	9.3185	22
23	0.50669	16.443	159.65	1.9735	32.452	0.06081	0.03081	9.7093	23
24	0.49193	16.935	170.97	2.0327	34.426	0.05905	0.02905	10.095	24
25	0.47761	17.413	182.43	2.0937	36.459	0.05743	0.02743	10.476	25
26	0.46369	17.876	194.02	2.1565	38.553	0.05594	0.02594	10.853	26
28	0.43708	18.764	217.53	2.2879	42.930	0.05329	0.02329	11.593	28
30	0.41199	19.600	241.36	2.4272	47.575	0.05102	0.02102	12.314	30
∞	0	33.333	1111.11	∞	∞	0.0300	0.0000	33.333	∞

$i = 4.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9615	0.9615	-0.0000	1.0400	1.0000	1.0400	1.0000	-0.0000	1
2	.9246	1.8861	0.9246	1.0816	2.0400	0.5302	0.4902	0.4902	2
3	.8890	2.7751	2.7025	1.1249	3.1216	0.3603	0.3203	0.9739	3
4	.8548	3.6299	5.2670	1.1699	4.2465	0.2755	0.2355	1.4510	4
5	.8219	4.4518	8.5547	1.2167	5.4163	0.2246	0.1846	1.9216	5
6	.7903	5.2421	12.5062	1.2653	6.6330	0.1908	0.1508	2.3857	6
7	.7599	6.0021	17.0657	1.3159	7.8983	0.1666	0.1266	2.8433	7
8	.7307	6.7327	22.1806	1.3686	9.2142	0.1485	0.1085	3.2944	8
9	.7026	7.4353	27.8013	1.4233	10.5828	0.1345	0.0945	3.7391	9
10	.6756	8.1109	33.8814	1.4802	12.0061	0.1233	0.0833	4.1773	10
11	.6496	8.7605	40.3772	1.5395	13.4864	0.1141	0.0741	4.6090	11
12	.6246	9.3851	47.2477	1.6010	15.0258	0.1066	0.0666	5.0343	12
13	.6006	9.9856	54.4546	1.6651	16.6268	0.1001	0.0601	5.4533	13
14	.5775	10.5631	61.9618	1.7317	18.2919	0.0947	0.0547	5.8659	14
15	.5553	11.1184	69.7355	1.8009	20.0236	0.0899	0.0499	6.2721	15
16	.5339	11.6523	77.7441	1.8730	21.8245	0.0858	0.0458	6.6720	16
17	.5134	12.1657	85.9581	1.9479	23.6975	0.0822	0.0422	7.0656	17
18	.4936	12.6593	94.3498	2.0258	25.6454	0.0790	0.0390	7.4530	18
19	.4746	13.1339	102.8933	2.1068	27.6712	0.0761	0.0361	7.8342	19
20	.4564	13.5903	111.5647	2.1911	29.7781	0.0736	0.0336	8.2091	20
21	.4388	14.0292	120.3414	2.2788	31.9692	0.0713	0.0313	8.5779	21
22	.4220	14.4511	129.2024	2.3699	34.2480	0.0692	0.0292	8.9407	22
23	.4057	14.8568	138.1284	2.4647	36.6179	0.0673	0.0273	9.2973	23
24	.3901	15.2470	147.1012	2.5633	39.0826	0.0656	0.0256	9.6479	24
25	.3751	15.6221	156.1040	2.6658	41.6459	0.0640	0.0240	9.9925	25
26	.3607	15.9828	165.1212	2.7725	44.3117	0.0626	0.0226	10.3312	26
28	.3335	16.6631	183.1424	2.9987	49.9676	0.0600	0.0200	10.9909	28
30	.3083	17.2920	201.0618	3.2434	56.0849	0.0578	0.0178	11.6274	30
∞	.0000	25.000	625.0000	∞	∞	0.0400	0.0000	25.0000	∞

TABLE 5.2. Compound Interest Factors (continued)

i = 5.00%

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9524	0.952	0.000	1.050	1.000	1.0500	1.0000	0.000	1
2	.9070	1.859	0.907	1.102	2.050	0.5378	0.4878	0.488	2
3	.8638	2.723	2.635	1.158	3.152	0.3672	0.3172	0.967	3
4	.8227	3.546	5.103	1.216	4.310	0.2820	0.2320	1.439	4
5	.7835	4.329	8.237	1.276	5.526	0.2310	0.1810	1.903	5
6	.7462	5.076	11.968	1.340	6.802	0.1970	0.1470	2.358	6
7	.7107	5.786	16.232	1.407	8.142	0.1728	0.1228	2.805	7
8	.6768	6.463	20.970	1.477	9.549	0.1547	0.1047	3.245	8
9	.6446	7.108	26.127	1.551	11.027	0.1407	0.0907	3.676	9
10	.6139	7.722	31.652	1.629	12.578	0.1295	0.0795	4.099	10
11	.5847	8.306	37.499	1.710	14.207	0.1204	0.0704	4.514	11
12	.5568	8.863	43.624	1.796	15.917	0.1128	0.0628	4.922	12
13	.5303	9.394	49.988	1.886	17.713	0.1065	0.0565	5.322	13
14	.5051	9.899	56.554	1.980	19.599	0.1010	0.0510	5.713	14
15	.4810	10.380	63.288	2.079	21.579	0.0963	0.0463	6.097	15
16	.4581	10.838	70.160	2.183	23.657	0.0923	0.0423	6.474	16
17	.4363	11.274	77.140	2.292	25.840	0.0887	0.0387	6.842	17
18	.4155	11.690	84.204	2.407	28.132	0.0855	0.0355	7.203	18
19	.3957	12.085	91.328	2.527	30.539	0.0827	0.0327	7.557	19
20	.3769	12.462	98.488	2.653	33.066	0.0802	0.0302	7.903	20
21	.3589	12.821	105.667	2.786	35.719	0.0780	0.0280	8.242	21
22	.3418	13.163	112.846	2.925	38.505	0.0760	0.0260	8.573	22
23	.3256	13.489	120.009	3.072	41.430	0.0741	0.0241	8.897	23
24	.3101	13.799	127.140	3.225	44.502	0.0725	0.0225	9.214	24
25	.2953	14.094	134.228	3.386	47.727	0.0710	0.0210	9.524	25
26	.2812	14.375	141.259	3.556	51.113	0.0696	0.0196	9.827	26
28	.2551	14.898	155.110	3.920	58.403	0.0671	0.0171	10.411	28
30	.2314	15.372	168.623	4.322	66.439	0.0651	0.0151	10.969	30
∞	0	20.000	400.000	∞	∞	.0500	0	20.000	∞

i = 6.00%

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9434	0.9434	-0.0000	1.0600	1.0000	1.0600	1.0000	-0.0000	1
2	.8900	1.8334	0.8900	1.1236	2.0600	0.5454	0.4854	0.4854	2
3	.8396	2.6730	2.5692	1.1910	3.1836	0.3741	0.3141	0.9612	3
4	.7921	3.4651	4.9455	1.2625	4.3746	0.2886	0.2286	1.4272	4
5	.7473	4.2124	7.9345	1.3382	5.6371	0.2374	0.1774	1.8836	5
6	.7050	4.9173	11.4594	1.4185	6.9753	0.2034	0.1434	2.3304	6
7	.6651	5.5824	15.4497	1.5036	8.3938	0.1791	0.1191	2.7676	7
8	.6274	6.2098	19.8416	1.5938	9.8975	0.1610	0.1010	3.1952	8
9	.5919	6.8017	24.5768	1.6895	11.4913	0.1470	0.0870	3.6133	9
10	.5584	7.3601	29.6023	1.7908	13.1808	0.1359	0.0759	4.0220	10
11	.5268	7.8869	34.8702	1.8983	14.9716	0.1268	0.0668	4.4213	11
12	.4970	8.3838	40.3369	2.0122	16.8699	0.1193	0.0593	4.8113	12
13	.4688	8.8527	45.9629	2.1329	18.8821	0.1130	0.0530	5.1920	13
14	.4423	9.2950	51.7128	2.2609	21.0151	0.1076	0.0476	5.5635	14
15	.4173	9.7122	57.5546	2.3966	23.2760	0.1030	0.0430	5.9260	15
16	.3936	10.1059	63.4592	2.5404	25.6725	0.0990	0.0390	6.2794	16
17	.3714	10.4773	69.4011	2.6928	28.2129	0.0954	0.0354	6.6240	17
18	.3503	10.8276	75.3569	2.8543	30.9057	0.0924	0.0324	6.9597	18
19	.3305	11.1581	81.3062	3.0256	33.7600	0.0896	0.0296	7.2867	19
20	.3118	11.4699	87.2304	3.2071	36.7856	0.0872	0.0272	7.6051	20
21	.2942	11.7641	93.1136	3.3996	39.9927	0.0850	0.0250	7.9151	21
22	.2775	12.0416	98.9412	3.6035	43.3923	0.0830	0.0230	8.2166	22
23	.2618	12.3034	104.7007	3.8197	46.9958	0.0813	0.0213	8.5099	23
24	.2470	12.5504	110.3812	4.0489	50.8156	0.0797	0.0197	8.7951	24
25	.2330	12.7834	115.9732	4.2919	54.8645	0.0782	0.0182	9.0722	25
26	.2198	13.0032	121.4684	4.5494	59.1564	0.0769	0.0169	9.3414	26
28	.1956	13.4062	132.1420	5.1117	68.5281	0.0746	0.0146	9.8568	28
30	.1741	13.7648	142.3588	5.7435	79.0582	0.0726	0.0126	10.3422	30
∞	.0000	16.6667	277.7778	∞	∞	0.0600	0.0000	16.6667	∞

TABLE 5.2. Compound Interest Factors (continued)

$i = 7\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	0.93458	0.9345	0.0000	1.0700	1.0000	1.07000	1.00000	0.0000	1
2	0.87344	1.8080	0.8734	1.1449	2.0700	0.55309	0.48309	0.4830	2
3	0.81630	2.6243	2.5060	1.2250	3.2149	0.38105	0.31105	0.9549	3
4	0.76290	3.3872	4.7947	1.3108	4.4399	0.29523	0.22523	1.4155	4
5	0.71299	4.1002	7.6466	1.4025	5.7507	0.24389	0.17389	1.8649	5
6	0.66634	4.7665	10.978	1.5007	7.1532	0.20980	0.13980	2.3032	6
7	0.62275	5.3892	14.714	1.6057	8.6540	0.18555	0.11555	2.7303	7
8	0.58201	5.9713	18.788	1.7181	10.259	0.16747	0.09747	3.1465	8
9	0.54393	6.5152	23.140	1.8384	11.978	0.15349	0.08349	3.5517	9
10	0.50835	7.0235	27.715	1.9671	13.816	0.14238	0.07238	3.9460	10
11	0.47509	7.4986	32.466	2.1048	15.783	0.13336	0.06336	4.3296	11
12	0.44401	7.9426	37.350	2.2521	17.888	0.12590	0.05590	4.7025	12
13	0.41496	8.3576	42.330	2.4098	20.140	0.11965	0.04965	5.0648	13
14	0.38782	8.7454	47.371	2.5785	22.550	0.11434	0.04434	5.4167	14
15	0.36245	9.1079	52.446	2.7590	25.129	0.10979	0.03979	5.7582	15
16	0.33873	9.4466	57.527	2.9521	27.888	0.10586	0.03586	6.0896	16
17	0.31657	9.7632	62.592	3.1588	30.840	0.10243	0.03243	6.4110	17
18	0.29586	10.059	67.621	3.3799	33.999	0.09941	0.02941	6.7224	18
19	0.27651	10.335	72.599	3.6165	37.379	0.09675	0.02675	7.0241	19
20	0.25842	10.594	77.509	3.8696	40.995	0.09439	0.02439	7.3163	20
21	0.24151	10.835	82.339	4.1405	44.865	0.09229	0.02229	7.5990	21
22	0.22571	11.061	87.079	4.4304	49.005	0.09041	0.02041	7.8724	22
23	0.21095	11.272	91.720	4.7405	53.436	0.08871	0.01871	8.1368	23
24	0.19715	11.469	96.254	5.0723	58.176	0.08719	0.01719	8.3923	24
25	0.18425	11.653	100.67	5.4274	63.249	0.08581	0.01581	8.6391	25
26	0.17220	11.825	104.98	5.8073	68.676	0.08456	0.01456	8.8773	26
28	0.15040	12.137	113.22	6.6488	80.697	0.08239	0.01239	9.3289	28
30	0.13137	12.409	120.97	7.6122	94.460	0.08059	0.01059	9.7486	30
∞	0	14.286	204.08	∞	∞	0.0700	0.0000	14.286	∞

$i = 8.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9259	0.9259	-0.0000	1.0800	1.0000	1.0800	1.0000	-0.0000	1
2	.8573	1.7833	0.8573	1.1664	2.0800	0.5608	0.4808	0.4808	2
3	.7938	2.5771	2.4450	1.2597	3.2464	0.3880	0.3080	0.9487	3
4	.7350	3.3121	4.6501	1.3605	4.5061	0.3019	0.2219	1.4040	4
5	.6806	3.9927	7.3724	1.4693	5.8666	0.2505	0.1705	1.8465	5
6	.6302	4.6229	10.5233	1.5869	7.3359	0.2163	0.1363	2.2763	6
7	.5835	5.2064	14.0242	1.7138	8.9228	0.1921	0.1121	2.6937	7
8	.5403	5.7466	17.8061	1.8509	10.6366	0.1740	0.0940	3.0985	8
9	.5002	6.2469	21.8081	1.9990	12.4876	0.1601	0.0801	3.4910	9
10	.4632	6.7101	25.9768	2.1589	14.4866	0.1490	0.0690	3.8713	10
11	.4289	7.1390	30.2657	2.3316	16.6455	0.1401	0.0601	4.2395	11
12	.3971	7.5361	34.6339	2.5182	18.9771	0.1327	0.0527	4.5957	12
13	.3677	7.9038	39.0463	2.7196	21.4953	0.1265	0.0465	4.9402	13
14	.3405	8.2442	43.4723	2.9372	24.2149	0.1213	0.0413	5.2731	14
15	.3152	8.5595	47.8857	3.1722	27.1521	0.1168	0.0368	5.5945	15
16	.2919	8.8514	52.2640	3.4259	30.3243	0.1130	0.0330	5.9046	16
17	.2703	9.1216	56.5883	3.7000	33.7502	0.1096	0.0296	6.2037	17
18	.2502	9.3719	60.8426	3.9960	37.4502	0.1067	0.0267	6.4920	18
19	.2317	9.6036	65.0134	4.3157	41.4463	0.1041	0.0241	6.7697	19
20	.2145	9.8181	69.0898	4.6610	45.7620	0.1019	0.0219	7.0369	20
21	.1987	10.0168	73.0629	5.0338	50.4229	0.0998	0.0198	7.2940	21
22	.1839	10.2007	76.9257	5.4365	55.4568	0.0980	0.0180	7.5412	22
23	.1703	10.3711	80.6726	5.8715	60.8933	0.0964	0.0164	7.7786	23
24	.1577	10.5288	84.2997	6.3412	66.7648	0.0950	0.0150	8.0066	24
25	.1460	10.6748	87.8041	6.8485	73.1059	0.0937	0.0137	8.2254	25
26	.1352	10.8100	91.1842	7.3964	79.9544	0.0925	0.0125	8.4352	26
28	.1159	11.0511	97.5687	8.6271	95.3388	0.0905	0.0105	8.8289	28
30	.0994	11.2578	103.4558	10.0627	113.2832	0.0888	0.0088	9.1897	30
∞	.0000	12.500	156.2500	∞	∞	0.0800	0.0000	12.5000	∞

TABLE 5.2. Compound Interest Factors (continued)

$i = 9\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	0.91743	0.9174	0.0000	1.0900	1.0000	1.09000	1.00000	0.0000	1
2	0.84168	1.7591	0.8416	1.1881	2.0900	0.56847	0.47847	0.4784	2
3	0.77218	2.5312	2.3860	1.2950	3.2781	0.39505	0.30505	0.9426	3
4	0.70843	3.2397	4.5113	0.4115	4.5731	0.30867	0.21867	1.3925	4
5	0.64993	3.8896	7.1110	1.5386	5.9847	0.25709	0.16709	1.8282	5
6	0.59627	4.4859	10.092	1.6771	7.5233	0.22292	0.13292	2.2497	6
7	0.54703	5.0329	13.374	1.8280	9.2004	0.19869	0.10869	2.6574	7
8	0.50187	5.5348	16.887	1.9925	11.028	0.18067	0.09067	3.0511	8
9	0.46043	5.9952	20.571	2.1718	13.021	0.16680	0.07680	3.4312	9
10	0.42241	6.4176	24.372	2.3673	15.192	0.15582	0.06582	3.7977	10
11	0.38753	6.8051	28.248	2.5804	17.560	0.14695	0.05695	4.1509	11
12	0.35553	7.1607	32.159	2.8126	20.140	0.13965	0.04965	4.4910	12
13	0.32618	7.4869	36.073	3.0658	22.953	0.13357	0.04357	4.8181	13
14	0.29925	7.7861	39.963	3.3417	26.019	0.12843	0.03843	5.1326	14
15	0.27454	8.0606	43.806	3.6424	29.360	0.12406	0.03406	5.4346	15
16	0.25187	8.3125	47.584	3.9703	33.003	0.12030	0.03030	5.7244	16
17	0.23107	8.5436	51.282	4.3276	36.973	0.11705	0.02705	6.0023	17
18	0.21199	8.7556	54.886	4.7171	41.301	0.11421	0.02421	6.2686	18
19	0.19449	8.9501	58.386	5.1416	46.018	0.11173	0.02173	6.5235	19
20	0.17843	9.1285	61.777	5.6044	51.160	0.10955	0.01955	6.7674	20
21	0.16370	9.2922	65.050	6.1088	56.764	0.10762	0.01762	7.0005	21
22	0.15018	9.4424	68.204	6.6586	62.873	0.10590	0.01590	7.2232	22
23	0.13778	9.5802	71.235	7.2578	69.531	0.10438	0.01438	7.4357	23
24	0.12640	9.7066	74.143	7.9110	76.789	0.10302	0.01302	7.6384	24
25	0.11597	9.8225	76.926	8.6230	84.700	0.10181	0.01181	7.8316	25
26	0.10639	9.9289	79.586	9.3991	93.324	0.10072	0.01072	8.0155	26
28	0.08955	10.116	84.541	11.167	112.96	0.09885	0.00885	8.3571	28
30	0.07537	10.273	89.028	13.267	136.30	0.09734	0.00734	8.6656	30
∞	0	11.111	123.45	∞	∞	0.0900	0.0000	11.111	∞

$i = 10.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9091	0.9091	-0.0000	1.1000	1.0000	1.1000	1.0000	-0.0000	1
2	.8264	1.7355	0.8264	1.2100	2.1000	0.5762	0.4762	0.4762	2
3	.7513	2.4869	2.3291	1.3310	3.3100	0.4021	0.3021	0.9366	3
4	.6830	3.1699	4.3781	1.4641	4.6410	0.3155	0.2155	1.3812	4
5	.6209	3.7908	6.8618	1.6105	6.1051	0.2638	0.1638	1.8101	5
6	.5645	4.3553	9.6842	1.7716	7.7156	0.2296	0.1296	2.2236	6
7	.5132	4.8684	12.7631	1.9487	9.4872	0.2054	0.1054	2.6216	7
8	.4665	5.3349	16.0287	2.1436	11.4359	0.1874	0.0874	3.0045	8
9	.4241	5.7590	19.4215	2.3579	13.5795	0.1736	0.0736	3.3724	9
10	.3855	6.1446	22.8913	2.5937	15.9374	0.1627	0.0627	3.7255	10
11	.3505	6.4951	26.3963	2.8531	18.5312	0.1540	0.0540	4.0641	11
12	.3186	6.8137	29.9012	3.1384	21.3843	0.1468	0.0468	4.3884	12
13	.2897	7.1034	33.3772	3.4523	24.5227	0.1408	0.0408	4.6988	13
14	.2633	7.3667	36.8005	3.7975	27.9750	0.1357	0.0357	4.9955	14
15	.2394	7.6061	40.1520	4.1772	31.7725	0.1315	0.0315	5.2789	15
16	.2176	7.8237	43.4164	4.5950	35.9497	0.1278	0.0278	5.5493	16
17	.1978	8.0216	46.5819	5.0545	40.5447	0.1247	0.0247	5.8071	17
18	.1799	8.2014	49.6395	5.5599	45.5992	0.1219	0.0219	6.0526	18
19	.1635	8.3649	52.5827	6.1159	51.1591	0.1195	0.0195	6.2861	19
20	.1486	8.5136	55.4069	6.7275	57.2750	0.1175	0.0175	6.5081	20
21	.1351	8.6487	58.1095	7.4002	64.0025	0.1156	0.0156	6.7189	21
22	.1228	8.7715	60.6893	8.1403	71.4027	0.1140	0.0140	6.9189	22
23	.1117	8.8832	63.1462	8.9543	79.5430	0.1126	0.0126	7.1085	23
24	.1015	8.9847	65.4813	9.8497	88.4973	0.1113	0.0113	7.2881	24
25	.0923	9.0770	67.6964	10.8347	98.3471	0.1102	0.0102	7.4580	25
26	.0839	9.1609	69.7940	11.9182	109.1818	0.1092	0.0092	7.6186	26
28	.0693	9.3066	73.6495	14.4210	134.2099	0.1075	0.0075	7.9137	28
30	.0573	9.4269	77.0766	17.4494	164.4940	0.1061	0.0061	8.1762	30
∞	.0000	10.0000	100.0000	∞	∞	0.1000	0.0000	10.0000	∞

TABLE 5.2. Compound Interest Factors (continued)

$i = 12.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.8929	0.8929	-0.0000	1.1200	1.0000	1.1200	1.0000	-0.0000	1
2	.7972	1.6901	0.7972	1.2544	2.1200	0.5917	0.4717	0.4717	2
3	.7118	2.4018	2.2208	1.4049	3.3744	0.4163	0.2963	0.9246	3
4	.6355	3.0373	4.1273	1.5735	4.7793	0.3292	0.2092	1.3589	4
5	.5674	3.6048	6.3970	1.7623	6.3528	0.2774	0.1574	1.7746	5
6	.5066	4.1114	8.9302	1.9738	8.1152	0.2432	0.1232	2.1720	6
7	.4523	4.5638	11.6443	2.2107	10.0890	0.2191	0.0991	2.5515	7
8	.4039	4.9676	14.4714	2.4760	12.2997	0.2013	0.0813	2.9131	8
9	.3606	5.3282	17.3563	2.7731	14.7757	0.1877	0.0677	3.2574	9
10	.3220	5.6502	20.2541	3.1058	17.5487	0.1770	0.0570	3.5847	10
11	.2875	5.9377	23.1288	3.4785	20.6546	0.1684	0.0484	3.8953	11
12	.2567	6.1944	25.9523	3.8960	24.1331	0.1614	0.0414	4.1897	12
13	.2292	6.4235	28.7024	4.3635	28.0291	0.1557	0.0357	4.4683	13
14	.2046	6.6282	31.3624	4.8871	32.3926	0.1509	0.0309	4.7317	14
15	.1827	6.8109	33.9202	5.4736	37.2797	0.1468	0.0268	4.9803	15
16	.1631	6.9740	36.3670	6.1304	42.7533	0.1434	0.0234	5.2147	16
17	.1456	7.1196	38.6973	6.8660	48.8837	0.1405	0.0205	5.4353	17
18	.1300	7.2497	40.9080	7.6900	55.7497	0.1379	0.0179	5.6427	18
19	.1161	7.3658	42.9979	8.6128	63.4397	0.1358	0.0158	5.8375	19
20	.1037	7.4694	44.9676	9.6463	72.0524	0.1339	0.0139	6.0202	20
21	.0926	7.5620	46.8188	10.8038	81.6987	0.1322	0.0122	6.1913	21
22	.0826	7.6446	48.5543	12.1003	92.5026	0.1308	0.0108	6.3514	22
23	.0738	7.7184	50.1776	13 5523	104.6029	0.1296	0.0096	6.5010	23
24	.0659	7.7843	51.6929	15.1786	118.1552	0.1285	0.0085	6.6406	24
25	.0588	7.8431	53.1046	17.0001	133.3339	0.1275	0.0075	6.7708	25
26	.0525	7.8957	54.4177	19.0401	150.3339	0.1267	0.0067	6.8921	26
28	.0419	7.9844	56.7674	23.8839	190.6989	0.1252	0.0052	7.1098	28
30	.0334	8.0552	58.7821	29.9599	241.3327	0.1241	0.0041	7.2974	30
∞	.0000	8.333	69.4444	∞	∞	0.1200	0.0000	8.3333	∞

$i = 15.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.8696	0.8696	-0.0000	1.1500	1.0000	1.1500	1.0000	-0.0000	1
2	.7561	1.6257	0.7561	1.3225	2.1500	0.6151	0.4651	0.4651	2
3	.6575	2.2832	2.0712	1.5209	3.4725	0.4380	0.2880	0.9071	3
4	.5718	2.8550	3.7864	1.7490	4.9934	0.3503	0.2003	1.3263	4
5	.4972	3.3522	5.7751	2.0114	6.7424	0.2983	0.1483	1.7228	5
6	.4323	3.7845	7.9368	2.3131	8.7537	0.2642	0.1142	2.0972	6
7	.3759	4.1604	10.1924	2.6600	11.0668	0.2404	0.0904	2.4498	7
8	.3269	4.4873	12.4807	3.0590	13.7268	0.2229	0.0729	2.7813	8
9	.2843	4.7716	14.7548	3.5179	16.7858	0.2096	0.0596	3.0922	9
10	.2472	5.0188	16.9795	4.0456	20.3037	0.1993	0.0493	3.3832	10
11	.2149	5.2337	19.1289	4.6524	24.3493	0.1911	0.0411	3.6549	11
12	.1869	5.4206	21.1849	5.3503	29.0017	0.1845	0.0345	3.9082	12
13	.1625	5.5831	23.1352	6.1528	34.3519	0.1791	0.0291	4.1438	13
14	.1413	5.7245	24.9725	7.0757	40.5047	0.1747	0.0247	4.3624	14
15	.1229	5.8474	26.6930	8.1371	47.5804	0.1710	0.0210	4.5650	15
16	.1069	5.9542	28.2960	9.3576	55.7175	0.1679	0.0179	4.7522	16
17	.0929	6.0472	29.7828	10.7613	65.0751	0.1654	0.0154	4.9251	17
18	.0808	6.1280	31.1565	12.3755	75.8364	0.1632	0.0132	5.0843	18
19	.0703	6.1982	32.4213	14.2318	88.2118	0.1613	0.0113	5.2307	19
20	.0611	6.2593	33.5822	16.3665	102.4436	0.1598	0.0098	5.3651	20
21	.0531	6.3125	34.6448	18.8215	118.8101	0.1584	0.0084	5.4883	21
22	.0462	6.3587	35.6150	21.6447	137.6316	0.1573	0.0073	5.6010	22
23	.0402	6.3988	36.4988	24.8915	159.2764	0.1563	0.0063	5.7040	23
24	.0349	6.4338	37.3023	28.6252	184.1678	0.1554	0.0054	5.7979	24
25	.0304	6.4641	38.0314	32.9190	212.7930	0.1547	0.0047	5.8834	25
26	.0264	6.4906	38.6918	37.8568	245.7120	0.1541	0.0041	5.9612	26
28	.0200	6.5335	39.8283	50.0656	327.1041	0.1531	0.0031	6.0960	28
30	.0151	6.5660	40.7526	66.2118	434.7451	0.1523	0.0023	6.2066	30
∞	.0000	6.6667	44.4444	∞	∞	0.1500	0.0000	6.6667	∞

TABLE 5.2. Compound Interest Factors (continued)

$i = 20.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.8333	0.8333	-0.0000	1.2000	1.0000	1.2000	1.0000	-0.0000	1
2	.6944	1.5278	0.6944	1.4400	2.2000	0.6545	0.4545	0.4545	2
3	.5787	2.1065	1.8519	1.7280	3.6400	0.4747	0.2747	0.8791	3
4	.4823	2.5887	3.2986	2.0736	5.3680	0.3863	0.1863	1.2742	4
5	.4019	2.9906	4.9061	2.4883	7.4416	0.3344	0.1344	1.6405	5
6	.3349	3.3255	6.5806	2.9860	9.9299	0.3007	0.1007	1.9788	6
7	.2791	3.6046	8.2551	3.5832	12.9159	0.2774	0.0774	2.2902	7
8	.2326	3.8372	9.8831	4.2998	16.4991	0.2606	0.0606	2.5756	8
9	.1938	4.0310	11.4335	5.1598	20.7989	0.2481	0.0481	2.8364	9
10	.1615	4.1925	12.8871	6.1917	25.9587	0.2385	0.0385	3.0739	10
11	.1346	4.3271	14.2330	7.4301	32.1504	0.2311	0.0311	3.2893	11
12	.1122	4.4392	15.4667	8.9161	39.5805	0.2253	0.0253	3.4841	12
13	.0935	4.5327	16.5883	10.6993	48.4966	0.2206	0.0206	3.6597	13
14	.0779	4.6106	17.6008	12.8392	59.1959	0.2169	0.0169	3.8175	14
15	.0649	4.6755	18.5095	15.4070	72.0351	0.2139	0.0139	3.9588	15
16	.0541	4.7296	19.3208	18.4884	87.4421	0.2114	0.0114	4.0851	16
17	.0451	4.7746	20.0419	22.1861	105.9306	0.2094	0.0094	4.1976	17
18	.0376	4.8122	20.6805	26.6233	128.1167	0.2078	0.0078	4.2975	18
19	.0313	4.8435	21.2439	31.9480	154.7400	0.2065	0.0065	4.3861	19
20	.0261	4.8696	21.7395	38.3376	186.6880	0.2054	0.0054	4.4643	20
21	.0217	4.8913	22.1742	46.0051	225.0256	0.2044	0.0044	4.5334	21
22	.0181	4.9094	22.5546	55.2061	271.0307	0.2037	0.0037	4.5941	22
23	.0151	4.9245	22.8867	66.2474	326.2369	0.2031	0.0031	4.6475	23
24	.0126	4.9371	23.1760	79.4968	392.4842	0.2025	0.0025	4.6943	24
25	.0105	4.9476	23.4276	95.3962	471.9811	0.2021	0.0021	4.7352	25
26	.0087	4.9563	23.6460	114.4755	567.3773	0.2018	0.0018	4.7709	26
28	.0061	4.9697	23.9991	164.8447	819.2233	0.2012	0.0012	4.8291	28
30	.0042	4.9789	24.2628	237.3763	1181.8816	0.2008	0.0008	4.8731	30
∞	.0000	5.0000	25.0000	∞	∞	0.2000	0.0000	5.0000	∞

$i = 25.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.8000	0.8000	0.0000	1.2500	1.0000	1.2500	1.0000	0.0000	1
2	.6400	1.4400	0.6400	1.5625	2.2500	0.6944	0.4444	0.4444	2
3	.5120	1.9520	1.6640	1.9531	3.8125	0.5123	0.2623	0.8525	3
4	.4096	2.3616	2.8928	2.4414	5.7656	0.4234	0.1734	1.2249	4
5	.3277	2.6893	4.2035	3.0518	8.2070	0.3718	0.1218	1.5631	5
6	.2621	2.9514	5.5142	3.8147	11.2588	0.3388	0.0888	1.8683	6
7	.2097	3.1611	6.7725	4.7684	15.0735	0.3163	0.0663	2.1424	7
8	.1678	3.3289	7.9469	5.9605	19.8419	0.3004	0.0504	2.3872	8
9	.1342	3.4631	9.0207	7.4506	25.8023	0.2888	0.0388	2.6048	9
10	.1074	3.5705	9.9870	9.3132	33.2529	0.2801	0.0301	2.7971	10
11	.0859	3.6564	10.8460	11.6415	42.5661	0.2735	0.0235	2.9663	11
12	.0687	3.7251	11.6020	14.5519	54.2077	0.2684	0.0184	3.1145	12
13	.0550	3.7801	12.2617	18.1899	68.7596	0.2645	0.0145	3.2437	13
14	.0440	3.8241	12.8334	22.7374	86.9495	0.2615	0.0115	3.3559	14
15	.0352	3.8593	13.3260	28.4217	109.6868	0.2591	0.0091	3.4530	15
16	.0281	3.8874	13.7482	35.5271	138.1085	0.2572	0.0072	3.5366	16
17	.0225	3.9099	14.1085	44.4089	173.6357	0.2558	0.0058	3.6084	17
18	.0180	3.9279	14.4147	55.5112	218.0446	0.2546	0.0046	3.6698	18
19	.0144	3.9424	14.6741	69.3889	273.5558	0.2537	0.0037	3.7222	19
20	.0115	3.9539	14.8932	86.7362	342.9447	0.2529	0.0029	3.7667	20
21	.0092	3.9631	15.0777	108.4202	429.6809	0.2523	0.0023	3.8045	21
22	.0074	3.9705	15.2326	135.5253	538.1011	0.2519	0.0019	3.8365	22
23	.0059	3.9764	15.3625	169.4066	673.6264	0.2515	0.0015	3.8634	23
24	.0047	3.9811	15.4711	211.7582	843.0329	0.2512	0.0012	3.8861	24
25	.0038	3.9849	15.5618	264.6978	1054.7912	0.2509	0.0009	3.9052	25
26	.0030	3.9879	15.6373	330.8722	1319.4890	0.2508	0.0008	3.9212	26
28	.0019	3.9923	15.7524	516.9879	2063.9515	0.2505	0.0005	3.9457	28
30	.0012	3.9950	15.8316	807.7936	3227.1743	0.2503	0.0003	3.9628	30
∞	.0000	4.0000	16.0000	∞	∞	0.2500	0.0000	4.0000	∞

Practice Problems

(If only a few are selected, choose those with a star.)

5.1 Which of the following would be most difficult to monetize?

a) maintenance cost
b) selling price
c) fuel cost
d) prestige
e) interest on debt

VALUE AND INTEREST

*5.2 If $1,000 is deposited in a savings account that pays 6% annual interest and all the interest is left in the account, what is the account balance after three years?

a) $840 b) $1,000 c) $1,180 d) $1,191 e) $3,000

*5.3 Your perfectly reliable friend, Merle, asks for a loan and promises to pay back $150 two years from now. If the minimum interest rate you will accept is 8%, what is the maximum amount you will loan Merle?

a) $119 b) $126 c) $129 d) $139 e) $150

EQUIVALENCE OF CASH FLOW PATTERNS

*5.4 The annual amount of a series of payments to be made at the end of each of the next twelve years is $500. What is the present worth of the payments at 8% interest compounded annually?

a) $500 b) $3,768 c) $6,000 d) $6,480 e) $6,872

*5.5 Consider a prospective investment in a project having a first cost of $300,000, operating and maintenance costs of $35,000 per year, and an estimated net disposal value of $50,000 at the end of thirty years. Assume an interest rate of 8%.

What is the present equivalent cost of the investment if the planning horizon is thirty years?

a) $670,000 b) $689,000 c) $720,000 d) $791,000 e) $950,000

If the project replacement will have the same first cost, life, salvage value, and operating and maintenance costs as the original, what is the capitalized cost of perpetual service?

a) $670,000 b) $689,000 c) $720,000 d) $765,000 e) infinite

*5.6 Maintenance expenditures for a structure with a twenty-year life will come as periodic outlays of $1,000 at the end of the fifth year, $2,000 at the end of the tenth year, and $3,500 at the end of the fifteenth year. With interest at 10%, what is the equivalent uniform annual cost of maintenance for the twenty-year period?

a) $200 b) $262 c) $300 d) $325 e) $342

5.7 After a factory has been built near a stream, it is learned that the stream occasionally overflows its banks. A hydrologic study indicates that the probability of flooding is about 1 in 8 in any one year. A flood would cause about $20,000 in damage to the factory. A levee can be constructed to prevent flood damage. Its cost will be $54,000 and its useful life is thirty years. Money can be borrowed for 8% interest. If the annual equivalent cost of the levee is less than the annual expectation of flood damage, the levee should be built. The annual expectation of flood damage is $(1/8) \times 20{,}000 = \$2{,}500$. Compute the annual equivalent cost of the levee.

a) $1,261 b) $1,800 c) $4,320 d) $4,800 e) $6,750

5.8 If $10,000 is borrowed now at 6% interest, how much will remain to be paid after a $3,000 payment is made four years from now?

a) $7,000 b) $9,400 c) $9,625 d) $9,725 e) $10,700

*5.9 A piece of machinery costs $20,000 and has an estimated life of eight years and a scrap value of $2,000. What uniform annual amount must be set aside at the end of each of the eight years for replacement if the interest rate is 4%?

a) $1,953 b) $2,170 c) $2,250 d) $2,500 e) $2,898

*5.10 The maintenance costs associated with a machine are $2,000 per year for the first ten years, and $1,000 per year thereafter. The machine has an infinite life. If interest is 10%, what is the present worth of the annual disbursements?

a) $16,145 b) $19,678 c) $20,000 d) $100,000 e) infinite

*5.11 A manufacturing firm entered into a ten-year contract for raw materials which required a payment of $100,000 initially and $20,000 per year beginning at the end of the fifth year. The company made unexpected profits and asked that it be allowed to make a lump sum payment at the end of the third year to pay off the remainder of the contract. What lump sum is necessary if the interest rate is 8%?

a) $85,600 b) $100,000 c) $120,000 d) $200,000 e) $226,000

UNUSUAL CASH FLOWS AND INTEREST PAYMENTS

5.12 A bank currently charges 10% interest compounded annually on business loans. If the bank were to change to continuous compounding, what would be the effective annual interest rate?

a) 10% b) 10.517% c) 12.5% d) 12.649% e) infinite

*5.13 Terry bought an electric typewriter for $50 down and $30 per month for 24 months. The same machine could have been purchased for $675 cash. What nominal annual interest rate is Terry paying?

a) 7.6% b) 13.9% c) 14.8% d) 15.2% e) 53.3%

5.14 How large a contribution is required to endow perpetually a research laboratory which requires $50,000 for original construction, $20,000 per year for operating expenses, and $10,000 every three years for new and replacement equipment? Interest is 4%.

a) $70,000 b) $640,000 c) $790,000 d) $1,000,000 e) infinite

ANNUAL EQUIVALENT COST COMPARISONS

*5.15 One of the two production units described below must be purchased. The minimum attractive rate of return is 12%. Compare the two units on the basis of equivalent annual cost.

	Unit A	Unit B
Initial Cost	$16,000	$30,000
Life	8 years	15 years
Salvage value	$ 2,000	$ 5,000
Annual operating cost	$ 2,000	$ 1,000

a) *A* — $5,058; *B* — $5,270
b) *A* — $4,916; *B* — $4,872
c) *A* — $3,750; *B* — $2,667
d) *A* — $1,010; *B* — $1,010
e) *A* — $2,676; *B* — $4,250

5.16 Tanks to hold a corrosive chemical are now being made of material *A*, and have a life of eight years and a first cost of $30,000. When these tanks are four years old, they must be relined at a cost of $10,000. If the tanks could be made of material *B*, their life would be twenty years and no relining would be necessary. If the minimum rate of return is 10%, what must be the first cost of a tank made of material *B* to make it economically equivalent to the present tanks?

a) $30,000 b) $40,000 c) $51,879 d) $58,760 e) $100,000

PRESENT EQUIVALENT COST COMPARISONS

5.17 Compute the life cycle cost of a reciprocating compressor with first cost of $120,000, annual maintenance cost of $9,000, salvage value of $25,000 and life of six years. The minimum attractive rate of return is 10%.

a) $120,000 b) $145,000 c) $149,000 d) $153,280 e) $167,900

5.18 A punch press costs $100,000 initially, requires $10,000 per year in maintenance expenses, and has no salvage value after its useful life of ten years. With interest at 10%, the capitalized cost of the press is:

a) $100,000 b) $161,400 c) $197,300 d) $200,000 e) $262,700

5.19 A utility is considering two alternatives for serving a new area. Both plans provide twenty years of service, but plan *A* requires one large initial investment, while plan *B* requires additional investment at the end of ten years. Neglect salvage value, assume interest at 8%, and determine the present cost of both plans.

	Plan A	Plan B
Initial Investment	$50,000	$30,000
Investment at end of 10 years	none	$30,000
Annual property tax and maintenance, years 1-10	$ 800	$ 500
Annual property tax and maintenance, years 11-20	$ 800	$ 900

a) *A* — $48,780; *B* — $49,250
b) *A* — $50,000; *B* — $30,000
c) *A* — $50,000; *B* — $60,000
d) *A* — $57,900; *B* — $50,000
e) *A* — $66,000; *B* — $74,000

*5.20 The heat loss of a bare steam pipe costs $206 per year. Insulation *A* will reduce heat loss by 93% and can be installed for $116; insulation *B* will reduce heat loss by 89% and can be installed for $60. The insulations require no additional expenses and will have no salvage value at the end of the pipe's estimated life of eight years. Determine the present net equivalent value of the two insulations if the interest rate is 10%.

a) *A* — $116; *B* — $90
b) *A* — $906; *B* — $918
c) *A* — $1,022; *B* — $978
d) *A* — $1,417; *B* — $1,406
e) *A* — $1,533; *B* — $1,467

INCREMENTAL APPROACH

5.21 A helicopter is needed for six years. Cost estimates for two copters are:

	The Whirl 2B	The Rote 8
Price	$95,000	$120,000
Annual maintenance	3,000	9,000
Salvage value	12,000	25,000
Life in years	3	6

With interest at ~~10%,~~ what is the annual cost advantage of the Rote 8?

a) 0 b) $4,260 c) $5,670 d) $5,834 e) $6,000

5.22 A standard prime mover costs $20,000 and has an estimated life of six years. By the addition of certain auxiliary equipment, an annual savings of $300 in operating costs can be obtained, and the estimated life of the prime mover extended to nine years. Salvage value in either case is $5,000. Interest on capital is 8%. Compute the maximum expenditure justifiable for the auxiliary equipment.

a) $1,149 b) $1,800 c) $2,700 d) $7,140 e) $13,300

*5.23 An existing electrical transmission line needs to have its capacity increased, and this can be done in either of two ways. The first method is to add a second conductor to each phase wire, using the same poles, insulators and fittings, at a construction cost of $15,000 per mile. The second method for increasing capacity is to build a second line parallel to the existing line, using new poles, insulators and fittings, at a construction cost of $23,000 per mile. At some time in the future, the transmission line will require another increase in capacity, with the first alternative now requiring a second line at a cost of $32,500 per mile, and the second alternative requiring added conductors at a cost of $23,000 per mile. If interest rate is 6%, how many years between the initial expenditure and the future expenditure will make the two methods economically equal?

a) 1 b) 3 c) 5 d) 10 e) 25

REPLACEMENT PROBLEMS

5.24 One year ago machine *A* was purchased at a cost of $2,000, to be useful for five years. However, the machine failed to perform properly and costs $200 per month for repairs, adjustments and shut-downs. A new machine *B* designed to perform the same functions is quoted at $3,500, with the cost of repairs and adjustments estimated to be only $50 per month. The expected life of machine *B* is five years. Except for repairs and adjustments, the operating costs of the two machines are substantially equal. Salvage values are insignificant. Using 8% interest rate, compute the incremental annual net equivalent value of machine *B*.

a) − $877 b) $923 c) $1,267 d) $1,800 $2,677

BREAK-EVEN ANALYSIS

5.25 Bear Air, an airline serving the Arctic, buys 6000 spark plugs per year at a price of $18 each. It costs them $85 to process each order. If Bear Air's minimum attractive rate of return is 10%, what is the most economic quantity of spark plugs to buy at one time?

a) 1 b) 115 c) 532 d) 561 e) 6,000

*5.26 Bear Air has been contracting its overhaul work to Aleutian Aeromotive for $40,000 per plane per year. Bear estimates that by building a $500,000 maintenance facility with a life of 15 years and a salvage value of $100,000, they could handle their own overhauls at a variable cost of only $30,000 per plane per year. The maintenance facility could be financed with a secured loan at 8% interest. What is the minimum number of plans Bear must operate in order to make the maintenance facility economically feasible?

a) 5 b) 6 c) 10 d) 40 e) 50

5.27 It costs Bear Air $1,200 to run a scheduled flight, empty or full, from Coldfoot to Frostbite. Moreover, each passenger generates a cost of $40. The regular ticket costs $90. The plane holds 65 people, but it is running only about 20 per flight. The sales director has suggested selling special tickets for $50 to Eskimos who do not normally fly.

What is the minimum number of Eskimo tickets that must be sold in order for a flight to produce a profit?

a) 5 b) 10 c) 15 d) 20 e) 45

What would be the total profit on the flight from Coldfoot to Frostbite if all 65 passengers started wearing sealskins and buying Eskimo tickets?

a) − $800 b) − $550 c) 0 d) $400 e) $500

5.28 Two electric motors are being considered for an application in which there is uncertainty concerning the hours of usage. Motor *A* costs $3,000 and has an efficiency of 90%. Motor *B* costs $1,400 and has an efficiency of 80%. Each motor has a ten-year life and no salvage value. Electric service costs $1.00 per year kW of demand and $0.01 per kWh of energy. The output of the motors is to be 74.6 kW, and interest rate is 8%. At how many hours usage per year would the two motors be equally economical? If the usage is less than this amount, which motor is preferable?

a) 1800, *A* b) 1800, *B* c) 2200, *A* d) 2200, *B* e) 2500, *A*

INCOME TAX AND DEPRECIATION

5.29 A drill press is purchased for $10,000 and has an estimated life of twelve years. The salvage value at the end of twelve years is estimated to be $1,300. Using straight-line depreciation, compute the book value of the drill press at the end of eight years.

a) $1,300 b) $3,333 c) $3,475 d) $4,200 e) $4,925

*5.30 A grading contractor owns earth-moving equipment that cost $90,000. At the end of its eight-year estimated life, it will have a salvage value of $18,000. Compute the depreciation for the first two years and the book value at the end of five years by the straight-line method.

a) $9,000; $9,000; $45,000
b) $11,250; $11,250; $33,750
c) $16,000; $14,000; $30,000
d) $18,000; $18,000; $90,000
e) $22,500; $16,875; $21,358

*5.31 Rework Problem 5.30 using the sum of years' digits method.

a) $9,000; $9,000; $45,000
b) $11,250; $11,250; $33,750
c) $16,000; $14,000; $30,000
d) $18,000; $18,000; $90,000
e) $22,500; $16,875; $21,358

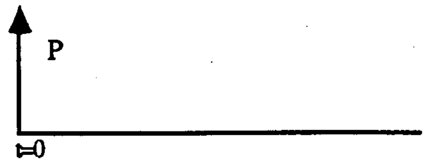
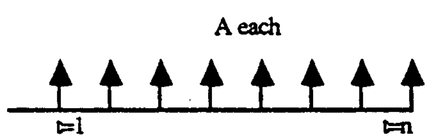
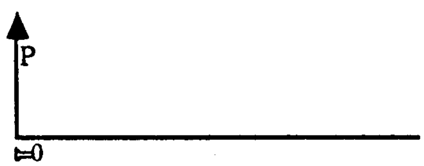
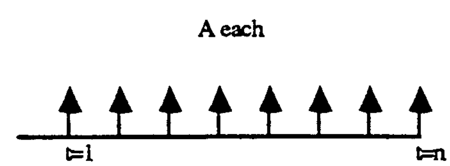
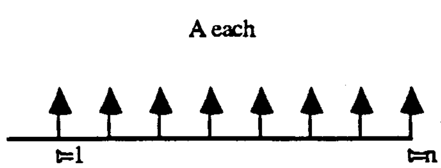
*5.32 Rework Problem 5.30 using the double-declining balance method.

a) $9,000; $9,000; $45,000
b) $11,250; $11,250; $33,750
c) $16,000; $14,000; $30,000
d) $18,000; $18,000; $90,000
e) $22,500; $16,875; $21,358

5.33 An asset has an initial cost of $80,000, an expected life of six years, and no salvage value. Assume a minimum attractive rate of return of ten percent and an incremental tax rate of fifty percent. Compute the present equivalent value of the tax savings achieved by using the sum of the years' digits method rather than the straight-line method for computing depreciation.

a) $2,300 b) $4,600 c) $29,000 d) $31,300 e) $58,000

Standard Cash Flow Factors

Multiply	by	To Obtain
 F	$1/(1+i)^n$	P 
 P	$(1+i)^n$	F 
 A	$\dfrac{(1+i)^n - 1}{i\ (1+i)^n}$	P 
 P	$\dfrac{i\ (1+i)^n}{(1+i)^n - 1}$	A 
 A	$\dfrac{(1+i)^n - 1}{i}$	F 
 F	$\dfrac{i}{(1+i)^n - 1}$	A